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Numerical investigation of the first bifurcation for natural convection of fluids enclosed in a 2D square cavity with Pr lower than 1.0

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ABSTRACT

This article presents a numerical study of the effect of the Prandtl number on the first bifurcation of natural convection for fluids enclosed in a 2D square cavity subject to a horizontal temperature gradient. The natural convection equations are solved using a second-order Euler–Taylor–Galerkin (ETG) finite element method of fractional steps. Influence of the mesh resolution on the numerical investigation is analyzed first on ten sets of uniform square element meshes while the Rayleigh numbers are 10^4 and 10^5 keeping Pr = 0.71. Variations of the averaged Nusselt number and its relative error in the results provided by the benchmark computation of Davis with the grids are used to find the role of mesh resolution. As for Ra = 10^4 and 10^5, Nu_{AVER} increases first with the increase in the number of grids used. And for each Ra, Nu_{AVER} tends to be independent of the number of elements when it is higher than 80×80. Grids (101×101) are then used in the study to capture the first bifurcation of natural convection. The bisection method and the flow patterns are utilized to estimate the critical Rayleigh number for 11 different fluids for which $Pr \leqslant 1.0$. It can be deduced from the results presented that Ra_{Cr} decreases with the increase in Pr. Variation of Ra_{Cr} with Pr is also fitted to estimate Ra_{Cr} for any fluids for which $Pr \leqslant 1.0$ directly. It is also observed that the global flow cores are inclined for each Pr and that the inclination degree increases in anticlockwise direction with Pr.

1. Introduction

Due to its importance in practical engineering applications and nonlinear sciences, natural convection in fluid-filled cavities has attracted a lot of interest in the past decades . Using a broad range of numerical methods, the buoyant flow has thoroughly been investigated either in 2D or 3D cavities [1–6]. What is well known is that the natural convection inside a cavity may make the evolution of a fluid system from a stationary state to a chaotic state through a series of bifurcations and transitions with the increase in the Rayleigh number. Variety of flow patterns with Ra experiences the same process for different fluids [6–8]. Previous studies have shown that the primary flow instability, a transition from diffusive thermal conduction to a stationary time-independent steady flow structure, occurs at critical Rayleigh numbers in the range of 10^3–10^4. And the value of Ra_C was found to be independent of the Prandtl number. However, this value depends on the cavity aspect ratio. Furthermore, with the increase in Ra, a Hopf bifurcation to a periodic oscillatory state is observed. Eventually, flow transitions to chaotic state occur with further increase in Ra. The corresponding Rayleigh numbers provided by different researchers varied in the range of approximately 10^6–10^8 [7–10]. Evolution of the flow system is characterized by a variety of flow patterns. For each fluid, variation of flow patterns with Ra experiences the same process. In fact, the phenomenon is intricate even for stationary states because of the interactions between the boundary layers and the flow cores. There are more than one global flow cores with the increase in Ra. Two cores are observed at Ra in the range of about 10^4–10^5. For Ra = 10^6, there have been three cores. Flow evolution has experienced two bifurcations, the first and the second. Though numerous works dealing with the topic of natural convection in closed cavities are available, none of them report on the estimation of the corresponding critical Rayleigh numbers for the first two bifurcations [1–6,8]. Moreover, in order to understand the phenomenon more clearly, investigation into the effect of the Prandtl number on the two bifurcations has yet to be undertaken successfully.

In the present paper, considering the mesh resolution effect, numerical investigations into the critical Rayleigh numbers for different fluids enclosed in a square cavity subject to a horizontal temperature gradient are carried out through a second-order Euler–Taylor–Galerkin (ETG) finite element method of fractional steps developed by the authors. Ten sets of uniform square element meshes were generated to find the effect of mesh resolution on the computation first. Configurations and typical mesh with

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elements are plotted in Fig. 1. In order to validate our scheme and code, numerical studies on natural convections for Ra = 10^4 and 10^5 with Pr = 0.71 are conducted, and the results are compared with those provided by Vaul G. Davis. Although all the results match well with those of Davis, the averaged Nusselt (Nu_{AVER}) number varies with the number of grids used in the computation. It increases first with the increase in the number of elements. As the grid number is higher than 81 × 81, Nu_{AVER} tends to be independent of the number of grids. And 101 × 101 grids are then used in the numerical simulation. Since the flow is stationary, flow patterns at different values of Ra and the bisection method are used to capture the first bifurcation of natural convection. Ra_{Cr} is defined as the onset of the second global flow core. The values of Ra_{Cr} for 10 different fluids with Pr $\leqslant$ 1.0 are computed to find the influence of Pr on the first bifurcation of natural convection. It can be deduced from the given results that Ra_{Cr} decreases with the increase in Pr. The curve of Ra_{Cr} vs. Pr is fitted. The equation can be used to estimate Ra_{Cr} for fluids for which Pr $\leqslant$ 1.0 directly. Moreover, in our computation, it is observed that the global cores are inclined irrespective of the number of the global cores. And the inclination degree increases in anticlockwise direction with the increase in Pr. For each Pr, the second flow core rounds up with Ra.

2. Mathematical formulation

Natural convections in a square cavity are governed by the differential equations representing the conservation of mass, momentum and energy. And the present study is based on the assumption that the flow is incompressible and two dimensional. In the momentum equation, the density factor alone is varied, whereas the thermal physical properties of the fluid in the flow model are kept constant.

The variables are given as follows:

$$x_i = \frac{x_i^*}{L}, \quad u_i = \frac{u_i^* L}{\alpha}, \quad p = \frac{p^* L^2}{\rho \alpha^2}, \quad t = \frac{t^* L^2}{\alpha}, \quad \theta = \frac{T - T_C}{T_H - T_C},$$

$$\mathrm{Pr} = \frac{\nu}{\alpha}, \quad \mathrm{Ra} = \frac{g\beta L^3 (T_H - T_C)}{\nu\alpha}$$

The governing equations of the 2D natural convection problem are represented in a non-dimensional form as

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mathrm{Pr}\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \mathrm{RaPr}\,\theta \cos\phi_i \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial \theta u_j}{\partial x_j} = \frac{\partial^2 \theta}{\partial x_j \partial x_j} \tag{3}$$

where the boundary conditions are as follows:

$$u(x,0) = u(x,1) = u(0,y) = u(1,y) = 0 \tag{4}$$

$$v(x,0) = v(x,1) = v(0,y) = v(1,y) = 0 \tag{5}$$

$$\theta(0,y) = 1, \quad \theta(1,y) = 0 \tag{6}$$

$$\frac{\partial \theta}{\partial y}(0,x) = \frac{\partial \theta}{\partial y}(1,x) = 0 \tag{7}$$

Here $x_i^*, i = 1,2$, are the distances measured along the horizontal and vertical directions, respectively; x_i, i = 1, 2, are corresponding dimensionless coordinates; $u_i^*, i = 1,2$, are the velocity components; u_i, i = 1, 2, are dimensionless velocity components; p^* and p are the pressure and the dimensionless pressure; ρ is the density; T_H and T_C are the temperature at hot and cold walls; θ is the dimensionless temperature; L is the side of the square cavity; Pr and Ra are Prandtl and Raleigh numbers, respectively; φ_i, i = 1, 2, are degrees between $\vec{g}$ and each coordinates direction.

The heat transfer coefficient in terms of the local Nusselt number is defined as

$$\mathrm{Nu}_l = -\frac{\partial \theta}{\partial n} \tag{8}$$

where n denotes the normal direction on a plane. The averaged Nusselt number is the average of local Nusselt number along the wall and is defined by the following equation: $\mathrm{Nu}_{AVER} = \frac{1}{S}\int_0^S (-\frac{\partial\theta}{\partial n}) dS$.

3. Method of solution

3.1. Discretization of the momentum equations

In this study, a second-order ETG finite element method of fractional steps is developed. First, for each velocity component u_i (i = 1, 2, u_1 = u, u_2 = v)

$$u_i^{n+1} = u_i^n + \Delta t \frac{\partial u_i^n}{\partial t} + \frac{\Delta t^2}{2}\frac{\partial^2 u_i^n}{\partial t^2} + O(\Delta t^3) \tag{9}$$

From continuity equation we have

$$\frac{\partial u_i^n}{\partial t} = \left(-u_j \frac{\partial u_i}{\partial x_j} - \frac{\partial p}{\partial x_i} + \mathrm{Pr}\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \mathrm{RaPr}\,\theta\cos\phi_i\right)^n \tag{10}$$

Substituting Eqs. (9) and (10)

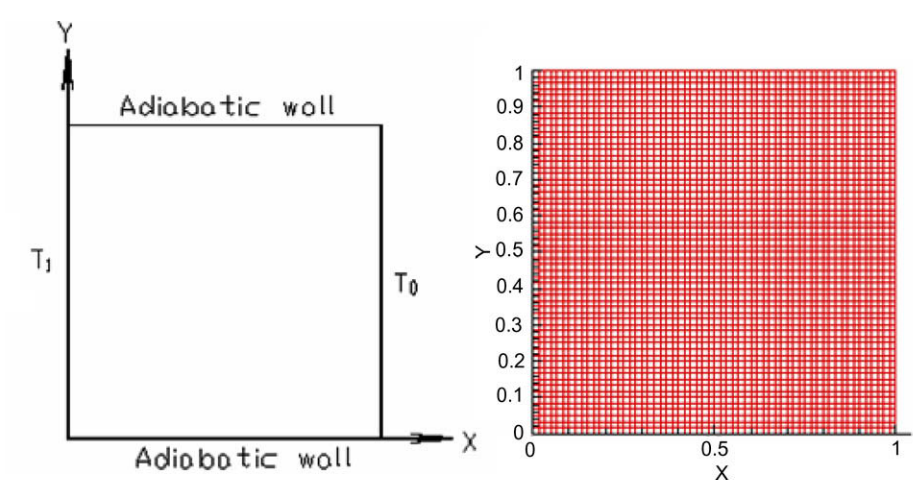

Fig. 1. Configurations and 60 × 60 uniform square elements.