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Performance and optimum design of convective–radiative rectangular fin with convective base heating, wall conduction resistance, and contact resistance between the wall and the fin base

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ABSTRACT

This paper investigates the performance and optimum design of a longitudinal rectangular fin attached to a convectively heated wall of finite thickness. The exposed surfaces of the fin lose heat to the environmental sink by simultaneous convection and radiation. The tip of the fin is assumed to lose heat by convection and radiation to the same sink. The analysis and optimization of the fin is conducted numerically using the symbolic algebra package Maple. The temperature distribution, the heat transfer rates, and the fin efficiency data is presented illustrating how the thermal performance of the fin is affected by the convection-conduction number, the radiation-conduction number, the base convection Biot number, the convection and radiation Biot numbers at the tip, and the dimensionless sink temperature. Charts are presented showing the relationship between the optimum convection-conduction number and the optimum radiation-conduction number for different values of the base convection Biot number and dimensionless sink temperature and fixed values of the convection and radiation Biot numbers at the tip. Unlike the few other papers which have applied the Adomian's decomposition and the differential quadrature element method to this problem but give illustrative results for specific fin geometry and thermal variables, the present graphical data are generally applicable and can be used by fin designers without delving into the mathematical details of the computational techniques.

1. Introduction

The performance and optimum design of longitudinal fins of rectangular profile with simultaneous surface convection and radiation have been studied in numerous publications because of the importance of such studies in many applications. A comprehensive review of the literature on this topic has been provided by Kraus et al. [1] and Aziz and Kraus [2] where numerous papers are cited and need not be repeated here. However, with the exception of few recent papers, the studies are based on the assumption of a constant and known fin base temperature. In many physical situations, the fin is attached to one side of a wall of finite thickness while the other side of the wall is in contact with a hot fluid from which heat transmitted through the wall is ultimately rejected by convection and radiation from the surface of the fin to the environment (sink). Aziz [3,4] found that the convection resistance of the hot fluid and the conduction resistance of the primary surface significantly affect the performance and optimum design of convecting fins of rectangular, triangular and concave parabolic profiles. Later Ma and Chung [5] adopted the same model and used a golden section search technique to establish optimum dimensions of convective–radiative rectangular fins. Subsequently, Chung and Zhou [6] extended the analysis in [5] to a radial (annular) fin of trapezoidal profile and included the contact resistance between the wall and the fin base in addition to the convective resistance of the hot fluid and the wall conduction resistance. More recently, Chiu and Chen [7] utilized Adomian's decomposition procedure to evaluate the heat transfer characteristics of a convecting–radiating longitudinal fin of rectangular profile. They improved upon the previously cited works by considering a convective–radiative fin tip (instead of an insulated fin tip) and allowing the thermal conductivity of the fin to vary with temperature. However, no optimization analysis was performed. The optimization study omitted by Chiu and Chen [7] was later conducted by Malekzadeh et al. [8] who used the differential quadrature element method (DQEM) in conjunction with the golden section search for the optimum design. To check the accuracy of the DQEM, they also used Adomian's decomposition procedure, Taylor transformation technique [9], and the finite difference method and claimed the DQEM was computationally superior to finite difference method. Chiu and Chen [7], Malekzadeh et al. [8] and Yu and Chen [9] all illustrate their procedures by citing and

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Nomenclature

a	thermal conductivity parameter, dimensionless
A_c	fin cross-sectional area, m^2
Bi	Biot number for base convection, hL/k, dimensionless
$N_{t,\,1}$	convection Biot number at fin tip, h_cL/k, dimensionless
$N_{t,\,2}$	radiation Biot number at fin tip, $\varepsilon\sigma LT_f^3/k$, dimensionless
h	lumped heat transfer coefficient, $\left[\frac{1}{h_f}+\frac{\delta_w}{k_w}+R''_{t,c}\right]^{-1}$
h_c	convection coefficient of the sink fluid, W/m^2 K
h_f	convection coefficient of the base fluid, W/m^2 K
h_r	radiation heat transfer coefficient, W/m^2 K
k	fin, thermal conductivity at temperature T, W/m K
k_s	fin thermal conductivity at temperature T_s, W/m K
k_f	thermal conductivity of the base fluid, W/m K
k_w	thermal conductivity of the wall, W/m K
L	fin length, m
Nc	convection-conduction number, h_cPL^2/k_sA_c, dimensionless
N^*	linearized radiation-conduction number, h_rPL^2/k_sA_c dimensionless
Nr	radiation-conduction number, $\varepsilon\sigma PL^2T_f^3/k_sA_c$, dimensionless
P	fin perimeter, m
q	fin heat transfer rate, W
q_{ideal}	ideal fin heat transfer rate, W
Q	dimensionless heat transfer rate, $qL/k_sA_cT_f$
T	fin temperature at any location x, K
T_b	fin base temperature, K
T_f	temperature of the base fluid, K
T_s	sink temperature, K
w	fin thickness, m
x	axial distance measured from the base of the fin, m
X	dimensionless distance, x/L
ε	fin surface emissivity, dimensionless
σ	Stefan–Boltzmann constant, 5.67×10^{-8} W/m^2 K^4
δ	wall thickness
θ	dimensionless temperature, T/T_f
θ_s	dimensionless sink temperature, T_s/T_f
opt	optimum

discussing the numerical results for a single fin with specified material properties, base convection thermal parameters, and environment conditions. Because of the specific nature of the results, the designers need to use these computational procedures for their own design calculations. Furthermore, none of the papers cited provide any information on the efficiency of the fin which is a quantity of fundamental interest.

The present work therefore has three objectives. The first is to modify the mathematical model in [7–9] to include the effects of wall conduction resistance and the contact resistance between the wall and the fin base. The second one is to generate performance and design charts that are general purpose and readily usable rather than the limited to one fin example as in [7–9]. The third objective is to demonstrate that such information can be easily generated (with a few lines of code) by using the symbolic algebra package Maple which offers a highly accurate and extensively tested fourth–fifth order Runge–Kutta–Fehlberg algorithm with automatic step size control [10] for solving nonlinear boundary value problems. The same Maple program can be used to derive the optimum fin design information.

2. Mathematical model

Consider a rectangular longitudinal fin of length L, thickness w, cross-sectional area A_c, perimeter P, thermal conductivity k and surface emissivity ε attached to a wall of thickness δ_w and thermal conductivity k_w as shown in Fig. 1. The contact resistance between the wall and the fin is $R''_{t,c}$. The rear side of the wall is in contact with a fluid at temperature T_f which heats the wall through a convective heat transfer coefficient h_f. The heat conducted through the base of the fin is dissipated by convection (characterized by a convection heat transfer coefficient h_c) and radiation to the environment. For both convective and radiative heat dissipations, the sink temperature is assumed to be the same, i.e. T_s. The tip of the fin also convects and radiates heat to the same sink. The fin is assumed to contain no internal heat sources although a source term can be easily incorporated in the differential equation if necessary.

The thermal conductivity of the fin k is assumed to vary linearly with temperature, that is,

$$k = k_s[1+\alpha(T-T_s)] \tag{1}$$

where α is a constant and k_s is the thermal conductivity at the sink temperature T_s. To keep reasonable number of variables in the analysis, the convection resistance between the hot fluid and the wall, the conduction resistance of the wall, and the contact resistance between the wall and the fin are lumped together by defining a convection heat transfer coefficient h as follows:

$$h = \left[\frac{1}{h_f}+\frac{\delta_w}{k_w}+R''_{t,c}\right]^{-1} \tag{2}$$

For one-dimensional conduction in the fin, the energy equation and the boundary conditions are as follows.

$$\frac{d}{dx}\left[k_s(1+a(T-T_s))\frac{dT}{dx}\right] - h_cP(T-T_s) - \varepsilon\sigma P(T^4-T_s^4) = 0 \tag{3}$$

$$x=0,\quad -k\frac{dT}{dx} = h(T_f-T) \tag{4}$$

$$x=L,\quad -k\frac{dT}{dx} = h_c(T-T_s)+\varepsilon\sigma(T^4-T_s^4) \tag{5}$$

The fin heat transfer rate q equals the heat conduction rate at the base of the fin which is

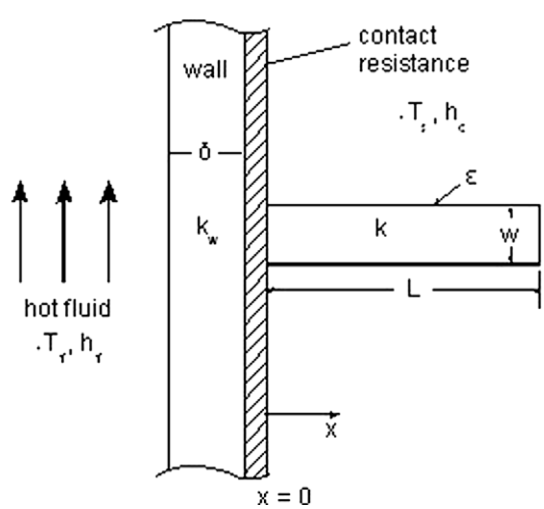

Fig. 1. Convecting–radiating fin with convective base heating.