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Optimum tilt angle of solar collectors for building applications in mid-latitude zone

S. Soulayman *, M. Hammoud

Higher Institute for Applied Sciences and Technology, Damascus, Syria

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ABSTRACT

For the middle latitudes (mid-latitude) zone (latitudes between 23.45°N and 43.45°N and between 23.45°S and 43.45°S), as rules of thumb first solar collector should be orientated toward Equator and second it should have a latitude tilt value; however are these statements valid all over the year? The present work focuses on presenting an algorithm for determining the optimum tilt angle over mid-latitude zone and for any collector azimuth angle. Moreover, two simple approximate equations are proposed for predicting daily optimum tilt angle and optimum tilt angle for any number of consecutive months. The present algorithm was applied at different latitudes where data are available. The different yearly possible energy gains in relation that received by a horizontal surface were calculated. It is found that the yearly daily average energy gain for daily, monthly, seasonally and half-yearly adjustments are approximately constant. For the latitude of 43.45°N it reaches 1.7 times that of horizontal surface. So, it is sufficient from practical point of view to adjust the solar collector tilt angle twice a year: once on 22/3 and the other on 22/9. Moreover, the first rule of thumb is valid however the second one is not applicable for a large number of consecutive days in the year.

1. Introduction

Domestic solar water heaters may supply 70–95% of the sanitary hot water in mid-latitude zone [1]. The majority of solar collectors are oriented with fixed mountings and the performance of thermal and photovoltaic modules and building-integrated systems is highly influenced by the modules' orientations. Therefore, it is often practicable to orient the solar collector at an optimum tilt angle, B_{opt} with respect to the local horizontal plane and to correct the tilt from time to time. In addition to direct solar applications, B_{opt} is crucial in the determination of the lengths of the building windows shading elements as well as in the right selection of their angles if they are located angular [1].

There are number of studies that were carried out with the aim of developing techniques to determine the optimum tilt angle of solar collectors around the world in the mid-latitude zone (Carbondale, Illinois [2], Izmir, Turkey [1], Sanliurfa, Turkey [3], Dhaka [4], several cities in China [5], Madinah, Saudi Arabia [6], Jordan [7], Helwan, Egypt [8], Cyprus [9], Burgos, Spain [10], Brisbane, Australia [11], Athens basin area [12], Mediterranean region [13], Ma'an, Jordan [14], Tabass, Iran [15], Hamirpur, India [16], South Africa [17], and many more). Therefore, it is of the great importance to be able to determine the optimum slope of the collector at any latitude, for any surface azimuth angle, and on any day or any period of the year in this zone. In this context, Soulayman [18] proposed a general algorithm for calculating B_{opt} for south facing collector. Furthermore, Soulayman and Sabbagh [19] proposed an algorithm which allowed the determination of B_{opt} at any latitude, L, and for any direction (surface azimuth angle, G). Stanciu and Stanciu [20] proposed a simple formula for determining the optimum tilt of south facing collector at latitudes from 0° to 80°. Nijegorodov et al. [21] presented 12 equations (one for each month), for determining optimum tilt angle for any location that lies between latitude 60°S to 60°N. Calabrò [22] proposed an algorithm to calculate the optimum tilt angle of solar panels by means of global horizontal solar radiation data, provided from Earth-based meteorological stations.

The objective of the present work is to present a modified general algorithm for treating B_{opt} over all mid-latitude zone and to shed a light on different suggested methods and provided results. The present work differs from those present in the literature by taking into account the solar intensity distribution in the algorithm of calculating daily optimum tilt angle.

* Corresponding author.
E-mail address: soulayman1952@gmail.com (S. Soulayman).

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Nomenclature

B	tilt angle (°) which is positive in NH and negative in SH
$B_{opt,d}$	daily optimum tilt angle (°)
$B_{opt,cd}$	characteristic day optimum tilt angle (°)
$B_{opt,hy}$	half-yearly optimum tilt angle (°)
$B_{opt,m}$	monthly optimum tilt angle (°)
$B_{opt,p}$	optimum tilt angle over a number of consecutive days (°)
$B_{opt,s}$	seasonally optimum tilt angle (°)
$B_{opt,y}$	yearly optimum tilt angle (°)
EF	equator facing
G	collector azimuth angle (°)
H_0	monthly average daily extraterrestrial solar radiation on a horizontal surface (MJ/m^2)
H	monthly average daily solar radiation on a horizontal surface (MJ/m^2)
H_t	monthly average daily solar radiation on a tilted surface (MJ/m^2)
$H_{t,d}$	daily solar radiation on a tilted surface (MJ/m^2)
K_T	monthly average clearness index
k_T	hourly average clearness index
L	latitude (°) which is positive at NH and negative at SH
N	north
n	day number in the year starting from January 1st
NH	Northern Hemisphere
n_1	day number of the beginning of the period
n_2	day number of the end of the period
R	energy gain factor of daily optimum tilted surface with relation to horizontal surface
R_1	energy gain factor of monthly optimum tilted surface with relation to horizontal surface
R_2	energy gain factor of seasonally optimum tilted surface with relation to horizontal one
R_3	energy gain factor of half-yearly optimum tilted surface with relation to horizontal one
R_4	energy gain factor of yearly optimum tilted surface with relation to horizontal one
R_5	energy gain factor of latitude tilted surface with relation to horizontal one
R_6	energy gain factor of vertical surface with relation to EF vertical one
R_b	geometric factor
R_H	relative humidity
S	south
s	daily sunshine duration (h)
S_0	day length (h)
SH	Southern Hemisphere
T	ambient temperature
W	hour angle (°) which is negative before solar noon and positive after solar noon
W_s	sunset hour angle on horizontal surface (rad)
W_{sr}	sunrise hour angle on tilted surface (rad)
W_{ss}	sunset hour angle on tilted surface (rad)

Greek letters

δ	solar declination angle (°)
ρ	diffuse reflectance of the surroundings
Θ	solar ray incident angle on tilted surface (°)
Θ_{noon}	noon solar ray incident angle (°)
Θ_z	solar ray incident angle on horizontal surface (°)
τ_b	atmospheric transmittance for beam radiation
τ_d	atmospheric transmittance for diffuse radiation

2. Methodology/algorithm

Radiation data are the best source of information for estimating average incident radiation. However, sometimes in the lack of such data, it is possible to apply empirical relationships by using data providing the ratio (s/S_0), where s is the monthly average daily hours of bright sunshine and S_0 is the day length of the average day of the month (the most popular and most commonly used model is known as the Angstrom-Prescott model [23]), the relative humidity (RH) [24] and the ambient temperature (T) [25], which are widely available from many hundreds of stations in most countries. The main part of empirical relationships is restricted to sunshine duration s. However, these relationships could be written as:

$$H = f\left(\frac{s}{S_0}, R_H, T\right)H_0 \tag{1}$$

where H = monthly average daily radiation on a horizontal surface and H_0 = monthly average daily extraterrestrial solar radiation on a horizontal surface. Taking into consideration that on Northern Hemisphere (NH) L is positive, tilt angle B is positive when surface is oriented toward Equator and negative for the opposite direction and on Southern Hemisphere (SH) L is negative, B is negative when surface is oriented toward Equator and positive for the opposite direction.

The monthly average daily total solar radiation H on a horizontal plane, provided from meteorological station, includes direct H_b and diffuse H_d components while the monthly average daily total solar radiation H_t on a tilted surface can be divided into direct, diffuse and ground reflected components. The H_d and H_b components of H can be estimated using empirical relationships H_d = f(H) by means of clearness index K_T [26]. If the diffuse and ground reflected H_g are each assumed to be isotropic, the total solar radiation on a tilted surface H_t, can be obtained as follows:

$$H_t = H\left(1 - \frac{H_d}{H}\right)R_b + H_d\left(\frac{1+\cos(B)}{2}\right) + \rho H\left(\frac{1-\cos(B)}{2}\right) \tag{2}$$

where ρ is the diffuse reflectance of the surroundings and R_b is geometric factor:

$$R_b = 0.5 \times \frac{\sum\{A_1(W_{ss} - W_{sr}) + A_2[\sin(W_{ss}) - \sin(W_{sr})] + A_3[\cos(W_{ss}) - \cos(W_{sr})]\}}{\sum\{W_s \sin(\delta)\sin(\mathrm{L}) + \cos(\delta)\sin(\mathrm{L})\sin(W_s)\}} \tag{3}$$

The angle of incidence of beam radiation on a tilted angle can be expressed using A_1, A_2, A_3 and solar hour angle W as follows:

$$\cos(\theta) = A_1 + A_2\cos(W) + A_3\sin(W) \tag{4}$$

$$A_1 = \sin(\delta)[\sin(L)\cos(B) - \sin(B)\cos(L)\cos(G)] \tag{5}$$

$$A_2 = \cos(\delta)[\cos(L)\cos(B) + \sin(B)\sin(L)\cos(G)] \tag{6}$$

$$A_3 = \cos(\delta)\sin(B)\sin(G); \tag{7}$$

δ (°) is the solar declination angle which could be calculated using the equation of Cooper [27]:

$$\delta = 23.45\sin\left[\frac{2\pi(n+284)}{365}\right] \tag{8}$$

W_{ss} (rad) is the sunset hour angle on a tilted surface: