Energy Conversion and Management 124 (2016) 247–265

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Review

# Review of criteria for the selection of probability distributions for wind speed data and introduction of the moment and L-moment ratio diagram methods, with a case study

T.B.M.J. Ouarda [a,b,*], C. Charron [b], F. Chebana [a]

[a] INRS-ETE, National Institute of Scientific Research, 490 de la Couronne, Quebec City (QC) G1K9A9, Canada
[b] Institute Center for Water and Environment (iWater), Masdar Institute of Science and Technology, P.O. Box 54224, Abu Dhabi, United Arab Emirates

ARTICLE INFO

*Article history:*
Received 16 February 2016
Received in revised form 12 June 2016
Accepted 4 July 2016
Available online 18 July 2016

*Keywords:*
Wind speed
Probability density distribution
Moment ratio diagram
L-moments
Goodness-of-fit criteria
Adequacy statistics

ABSTRACT

This paper reviews the different criteria used in the field of wind energy to compare the goodness-of-fit of candidate probability density functions (pdfs) to wind speed records, and discusses their advantages and disadvantages. The moment ratio and L-moment ratio diagram methods are also proposed as alternative methods for the choice of the pdfs. These two methods have the advantage of allowing an easy comparison of the fit of several pdfs for several time series (stations) on a single diagram. Plotting the position of a given wind speed data set in these diagrams is instantaneous and provides more information than a goodness-of-fit criterion since it provides knowledge about such characteristics as the skewness and kurtosis of the station data set. In this paper, it is proposed to study the applicability of these two methods for the selection of pdfs for wind speed data. Both types of diagrams are used to assess the fit of the pdfs for wind speed series in the United Arab Emirates. The analysis of the moment ratio diagrams reveals that the Kappa, Log-Pearson type III and Generalized Gamma are the distributions that fit best all wind speed series. The Weibull represents the best distribution among those with only one shape parameter. Results obtained with the diagrams are compared with those obtained with goodness-of-fit statistics and a good agreement is observed especially in the case of the L-moment ratio diagram. It is concluded that these diagrams can represent a simple and efficient approach to be used as complementary method to goodness-of-fit criteria.



**Contents**



* Corresponding author at: Institute Center for Water and Environment (iWater), Masdar Institute of Science and Technology, P.O. Box 54224, Abu Dhabi, United Arab Emirates.
*E-mail address:* touarda@masdar.ac.ae (T.B.M.J. Ouarda).

http://dx.doi.org/10.1016/j.enconman.2016.07.012
0196-8904/

**Nomenclature**

| | |
|---|---|
| $b_r$ | unbiased estimator of $B_r$ |
| $B_r$ | $r$th probability weighted moment where $M_{1,r,0}$ |
| $\beta_1$ | moment ratio $C_S^2$ |
| $\beta_2$ | moment ratio $C_K$ |
| $C_V$ | coefficient of variation |
| $C_S$ | coefficient of skewness |
| $C_K$ | coefficient of kurtosis |
| cdf | cumulative distribution function |
| $\chi^2$ | chi-square test statistic |
| D/M | distribution/method |
| EV1 | Gumbel or extreme value type I distribution |
| $f_{\hat{\theta}}()$ | probability density function with estimated parameters $\hat{\theta}$ |
| $\hat{f}()$ | estimated probability density function |
| $F_i$ | empirical probability for the $i$th wind speed observation |
| $\widehat{F}_i$ | estimated cumulative probability for the $i$th observation obtained with the theoretical cdf |
| $F()$ | cumulative distribution function |
| $F^{-1}()$ | inverse of a given cumulative distribution function |
| G | Gamma distribution |
| GEV | generalized extreme value distribution |
| GG | generalized Gamma distribution |
| GMM | generalized method of moment |
| KAP | Kappa distribution |
| KS | Kolmogorov-Smirnov test statistic |
| $\ell_{r+1}$ | sample $r$th L-moment |
| LM | method of L-moments |
| LN2 | 2-parameter Lognormal distribution |
| LN3 | 3-parameter Lognormal distribution |
| LP3 | Log-Pearson type III |
| $m_r$ | $r$th sample central moment |
| $M_{p,r,s}$ | probability weighted moment of order $p$, $r$, $s$ |
| ML | maximum likelihood |
| MM | method of moments |
| $\mu_r$ | $r$th central moment |
| $n$ | number of wind speed observations in a series of wind speed observations |
| $N$ | number of bins in a histogram of wind speed data |
| $p_i$ | the relative frequency at the $i$th class interval |
| $\hat{p}_i$ | the estimated probability at the $i$th class interval |
| $\widehat{\overline{P}}_0$ | mean wind power density for the theoretical pdf $f(v)$ |
| $\overline{P}_0$ | mean wind power density calculated from the observed wind speed data |
| $\widehat{\overline{P}}_w$ | mean wind turbine power from the theoretical pdf $f(v)$ |
| $\overline{P}_w$ | mean wind turbine power from the observed wind speed data |
| P3 | Pearson type III distribution |
| pdf | probability density function |
| $R^2$ | coefficient of determination |
| $R_a^2$ | adjusted $R^2$ |
| $R_{PP}^2$ | coefficient of determination giving the degree of fit between the theoretical cdf and the empirical cumulative probabilities of wind speed data |
| $R_{QQ}^2$ | coefficient of determination giving the degree of fit between the theoretical wind speed quantiles and the wind speed data |
| RMSE | root mean square error |
| $\tau_r$ | $r$th L-moment ratio |
| $t_r$ | $r$th sample L-moments ratio |
| $v_i$ | the $i$th observation of the wind speed series |
| $\hat{v}_i$ | predicted wind speed for the $i$th observation |
| W2 | 2-parameter Weibull distribution |
| W3 | 3-parameter Weibull distribution |



## 1. Introduction

The assessment of wind energy potential at a given site is often based on the use of probability density functions (pdfs) to characterize short term wind speed observations [1–16]. The selection of the appropriate pdf to model wind speed data is crucial in wind power energy applications as it reduces wind power output estimation uncertainties. Traditionally, the two-parameter Weibull (W2) is the most used pdf in studies related to wind speed data analysis [17]. While being extensively used in studies dedicated to the assessment of wind energy [18–25], the Weibull is not able to represent every wind speed regime [26–28]. Recently, a number of studies have used a variety of other pdfs with variable levels of success [17,22,27–40]. The pdfs used include the Gamma (G), Inverse Gamma (IG), Inverse Gaussian (IGA), two and three-parameter Lognormal (LN2, LN3), Logistic (L), Log-logistic (LL), Gumbel (EV1), Generalized Extreme Value (GEV), three-parameter Beta (B), Pearson type III (P3), Log-Pearson type III (LP3), Burr (BR), Erlang (ER), Kappa (KAP) and Wakeby (WA) distributions. Ouarda et al. [27] found the GG and KAP to be superior to W2 in the United Arab Emirates (UAE). Mert and Karakuş [34] found the Burr distribution to be more suitable than the GG or W2 for wind speed data in Antakya, Turkey.

A number of authors have proposed mixture distributions [13,27,28,31,41–46]. The mixture models were found to provide better fit in the case of distributions presenting bimodal characteristics. A model composed of two Weibull distributions is most often used [27,31,46–48]. Other mixture models used are the Normal-Normal, Truncated Normal-Weibull and Gamma-Weibull. Shin et al. [28] applied a large number of different mixture models to wind speed data in the UAE and concluded that the Weibull-Extreme value type-1 is the most appropriate distribution. The use of distributions generated by the maximum entropy principle is also common [13,49–52]. These distributions have the advantage