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Natural double-diffusive convection in a shallow horizontal rectangular cavity uniformly heated and salted from the side and filled with non-Newtonian power-law fluids: The cooperating case

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ABSTRACT

This paper reports an analytical and numerical study of double-diffusive natural convection in a non-Newtonian power-law fluid contained in a horizontal rectangular enclosure submitted to uniform heat and mass fluxes along its short vertical sides, while the horizontal ones are insulated and impermeable. The first part from this study is devoted to the numerical solution of the governing equations, and the effect of the governing parameters, namely, the cavity aspect ratio, A, the Lewis number, Le, the buoyancy ratio, N, the power-law behavior index, n, and the generalized Prandtl, Pr, thermal Rayleigh, Ra_T, numbers, is examined. In the second part, an analytical solution, based on the parallel flow approximation in the case of a shallow cavity ($A \gg 1$), is proposed and a good agreement is found between the two types of solutions.

1. Introduction

Double-diffusive natural convection, i.e. flows generated by buoyancy due to simultaneous temperature and concentration gradients, can be found in wide range of situations. In nature, such flows are encountered in the oceans, lakes, solar ponds, shallow coastal waters and the atmosphere. In industry, examples include chemical processes, crystal growth, energy storage, material and food processing, etc. For a review of the fundamental works in this area, see, for instance [1,2].

The literature related to natural double-diffusive convection shows that the majority of analytical, numerical and experimental investigations were focused on the enclosures of rectangular form. On this subject, the books of Bejan [3], Platten and Legros [4] and Nield and Bejan [5] constitute basic references.

In the past, many studies concerning Newtonian fluid flows in porous layers and fluid-filled cavities, driven simultaneously by thermal and solutal buoyancy effects, were carried out. A literature review reveals that studies on double-diffusive convection in enclosures can be classified under three types, according to their thermal and solutal boundary conditions. In the first type, the cavity is subjected to a vertical solutal gradient and a horizontal thermal one. For this situation, both experimental [6,7] and numerical [8,9] results show the formation of multi-layered roll cells separated by near-horizontal shape interfaces. In addition, the existence of multiple steady state solutions is possible [10] for a given set of the governing parameters. In the second type, both the temperature and concentration gradients are imposed transversally [11,12]. In such a case, as was observed for a porous layer [11], there exists a region in the plane (N = buoyancy ratio, Le = Lewis number) where the convective flow is not possible regardless of the Rayleigh, R_T, and Darcy, Da, numbers values. For a fluid-filled cavity [12], the onset of thermosolutal convection was studied, using Galerkin and finite element methods, and the thresholds for finite-amplitude, oscillatory and monotonic convection instabilities were determined explicitly in terms of the governing parameters. In diffusive mode, where solute is stabilizing, it was demonstrated that, when the thermal to solutal diffusivity ratio is greater than unity, overstability and subcritical convection may set in at a value of Ra_T well below the threshold of monotonic instability. In an infinite layer with rigid boundaries, the wavelength, at the onset of overstability, was found to be a function of the governing parameters. Analytical solutions, for finite-amplitude convection, were derived on the basis of a weak nonlinear perturbation theory, for general cases, and on the basis of the parallel flow approximation, for a shallow enclosure subject to Neumann boundary conditions. The stability of the parallel flow solution was studied and the threshold for Hopf bifurcation was determined. For a relatively large enclosure aspect ratio, the

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Nomenclature

A	aspect ratio of the cavity, Eq. (11)
C_T	dimensionless temperature gradient in the x-direction
C_S	dimensionless concentration gradient in the x-direction
D	mass diffusivity (m^2/s)
g	gravitational acceleration (m/s^2)
H'	height of the enclosure (m)
j'	constant mass flux per unit area (kg/m^2 s)
k	consistency index for a power-law fluid at the reference temperature (Pa s^n)
Le	lewis number, Eq. (11)
L'	length of the rectangular enclosure (m)
N	buoyancy ratio, Eq. (11)
n	flow behavior index for a power-law fluid at the reference temperature
Nu	local Nusselt number, Eqs. (12), (13) and (33)
$\overline{Nu}$	average Nusselt number, Eqs. (14) and (33)
Pr	generalised Prandtl number, Eq. (11)
q'	constant heat flux per unit area (W/m^2)
Ra_T	generalized thermal Rayleigh number, Eq. (11)
S	dimensionless concentration $[= (S' - S'_c)/\Delta S^*]$
S'_c	reference concentration at the geometric center of the enclosure (kg/m^3)
Sh	local Sherwood number, Eqs. (12), (13) and (33)
$\overline{Sh}$	mean Sherwood number, Eqs. (14) and (33)
T	dimensionless temperature, $[= (T' - T'_c)/\Delta T^*]$
T'_c	reference temperature at the geometric center of the enclosure (K)
ΔT^*	characteristic temperature [$=q'H'/\lambda$] (K)
ΔS^*	characteristic concentration [$=j'H'/D$] (kg/m^3)
(u,v)	dimensionless axial and vertical velocities [$=(u',v')(\alpha/H')$]
(x,y)	dimensionless axial and vertical co-ordinates [$=(x',y')/H'$]

Greek symbols

α	thermal diffusivity of fluid at the reference temperature (m^2/s)
β_T	thermal expansion coefficient of fluid at the reference temperature (1/K)
β_S	solutal expansion coefficient of fluid at the reference concentration (m^3/kg)
λ	thermal conductivity of fluid at the reference temperature (W/m K)
μ	dynamic viscosity for a Newtonian fluid at the reference temperature (Pa s)
μ_a	dimensionless apparent viscosity of fluid, Eq. (7)
ρ	density of fluid at the reference temperature (kg/m^3)
Ω	dimensionless vorticity, [$=\Omega'/(\alpha/H'^2)$]
ψ	dimensionless stream function, [$=\psi'/\alpha$]

Superscript

$'$	dimensional variable

Subscripts

c	value relative to the centre of the enclosure $(x,y) = (A/2, 1/2)$
*	characteristic variable

numerical solution indicates horizontally travelling waves developing near the threshold of the oscillatory convection. Multiple confined steady and unsteady states were found to coexist. In the third type, both the thermal and solutal gradients are imposed laterally [13–15]. For a vertical fluid-filled enclosure [13], in the first part of the analytical study, a scale analysis was applied to the two extreme cases of heat and mass-transfer-driven flows, while in the second part a parallel flow solution was reported for tall enclosures. Solutions for the flow, temperature and concentration fields and Nusselt and Sherwood numbers were obtained in terms of the problem governing parameters. In the limits of heat and solute-driven flows a good agreement was obtained between the predictions of the scale analysis and the analytical solution. The numerical solution of the complete governing equations, for the two-dimensional flow, was found to agree well with the analytical one. When the tall vertical cavity is a Darcy porous layer [14], a numerical study was performed to validate the results of analytical predictions. Hence, it was demonstrated in the case of opposing flows ($N < 0$) that there exists a domain in (Lewis = Le, Buoyancy ratio = N) plane where, at large values of Ra_T, boundary layer profiles are obtained for the velocity and the density but not for the temperature and the concentration. For a horizontal shallow cavity filled with a binary fluid [15], an analytical and numerical study revealed, in the opposing case ($N = -1$), the possibility of a steady rest state solution corresponding to a purely diffusive regime. Moreover, the existence of multiple solutions, for a given set of the governing parameters, was demonstrated both analytically and numerically for the values of N close to -1.

To our knowledge, for non-Newtonian fluids, except the work performed by Benhadji and Vasseur [16] in the case of a porous horizontal rectangular layer, where thermosolutal convection is generated inside a power-law fluid by application of horizontal or vertical uniform heat and mass fluxes, there is no investigations dealing with fluid-filled enclosures. These authors examined, by both numerical and analytical parallel flow approaches, the effect of the governing parameters, in particular that of the power-law behavior index, n, on the flow, temperature and concentration fields, and on the resulting heat and mass transfers. They observed that the shear-thinning behavior enhances the thermosolutal convection while the shear-thickening one reduces it, and that the results of the two approaches agree perfectly.

Otherwise, the majority of investigations concerning non-Newtonian fluids dealt with thermal driven buoyancy convection. In this respect, it is advisable to mention a recent work by Lamsaadi et al. [17], where natural convection, generated by imposing a lateral uniform heat flux to a horizontal slender rectangular enclosure confining non-Newtonian Ostwald–De Waele fluids, was studied by both numerical and analytical parallel flow ways. It was observed that the flow and temperature fields and the resulting thermal exchange are rather sensitive to non-Newtonian behavior than to Prandtl number variations, for its great values ($Pr \geqslant 100$). Furthermore, the analytical and numerical results were found to validate each other in the range of the governing parameters explored values.

In order to contribute to fill this gap, at least partly, the present study focuses on natural double-diffusive convection problem inside a two-dimensional horizontal rectangular enclosure filled with a non-Newtonian fluid. The cavity is submitted to uniform heat and mass fluxes from its short vertical sides, while its long horizontal boundaries are insulated and impermeable. The power-law model, suggested originally by Ostwald–De Waele, is adopted to characterize the non-Newtonian fluid behavior. In what follows, a numerical solution of the full governing equations is obtained for a wide range of the governing parameters, whose influence is amply discussed. In addition, an analytical solution, valid for stratified flows in slender enclosures, is derived on the basis of the parallel flow concept.