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Inverse problem of estimating transient heat transfer rate on external wall of forced convection pipe

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ABSTRACT

In this study, a conjugate gradient method based inverse algorithm is applied to estimate the unknown space and time dependent heat transfer rate on the external wall of a pipe system using temperature measurements. It is assumed that no prior information is available on the functional form of the unknown heat transfer rate; hence, the procedure is classified as function estimation in the inverse calculation. The accuracy of the inverse analysis is examined by using simulated exact and inexact temperature measurements. Results show that an excellent estimation of the space and time dependent heat transfer rate can be obtained for the test case considered in this study.

1. Introduction

Heat exchangers have long been one of the centerpieces in the fields of fluid mechanics and heat transfer due to its important applications in many industrial and daily life devices. Numerous review articles on the studies of heat exchangers can be found in the open literature, for example, Refs. [1–4]. Yet, research on heat exchangers has been intensified in recent years due to the heightened concern on energy conservation, which became particularly important in the fight against global warming. A focal point for heat exchanger research is heat transfer enhancement [1,2,5–7], which is one of the key issues on improving heat exchanger performance and producing more cost effective thermal systems. Successful applications of heat transfer enhancement techniques on heat exchangers can result in substantial energy savings and produce smaller systems to meet the required loads, saving both the construction and operation costs of the heat exchangers. In any heat exchanger design or analysis process, accurate measurement of the heat transfer rate between the hot and cold fluids is very important to evaluate the overall heat transfer coefficient, which is an essential parameter to assess the performance of the heat exchanger [8]. Despite there being many different flow arrangements, such as counter flow, parallel flow and cross flow arrangements in different types of heat exchangers, the hot and cold fluids in any heat exchanger are normally separated by solid material like a pipe wall where heat exchange takes place. Therefore, the heat transfer rate between hot and cold fluids can be equivalently obtained by estimating the heat flux through the solid/fluid boundaries of either the hot or cold fluids. Subsequently, the overall heat transfer rate of the entire system can be calculated, thus allowing the performance assessment for the heat exchanger to be conducted.

In recent years, studies of the inverse heat conduction problem (IHCP) have offered methods that largely scale down the experimental work to obtain accurate thermal quantities such as heat sources, material's thermal properties and boundary temperature or heat flux distributions in many heat conduction problems [9–12]. While there have been many reports on the IHCP, there are relatively fewer studies on inverse heat conduction–convection problems, presumably due to the complex nature of the latter [13–15]. Yet, there have been even fewer studies on the inverse heat transfer problem involving conjugate heat transfer. Chen et al. [16,17] reported inverse studies on estimating the inlet temperature and wall heat flux of laminar and turbulent pipe flow involving conjugate heat transfer. The techniques used were the function specification method and the least square error method, which require no iteration procedure. However, the estimated quantities were limited to being time dependent. Zueco and Alhama [18] recently proposed a new inverse procedure to estimate the temperature dependent thermal properties of a fully developed flow in a circular pipe. A time dependent heat flux was applied on the external wall of the pipe. Thus, the problem covers both the solid and fluid domains and involves conjugate heat transfer.

In this study, a pipe system similar to that of Ref. [18] is considered. The heat flux through a pipe's external wall, which is virtually

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Nomenclature

D_1, D_2	length parameters (m)
E	thickness of wall (m)
J	functional
J'	gradient of functional
k	thermal conductivity (W m^{-1} K^{-1})
L	length of duct (m)
M	total number of measuring positions
p	direction of descent
q	heat flux at wall (W m^{-2})
R	inner radius of duct (m)
r	spatial radial coordinate (m)
T	temperature (K)
T_0	initial temperature (K)
t	time (s)
u	axial fluid velocity (m s^{-1})
x	spatial axial coordinate (m)
Y	measurement temperature (K)
Δ	small variation quality
α	thermal diffusivity (m^2 s^{-1})
β	step size
γ	conjugate coefficient
η	very small value
λ	variable used in adjoint problem
σ	standard deviation
τ	transformed time (s)
ϖ	random variable
Superscripts	
K	iterative number
Subscripts	
f	fluid
s	solid

the rate of heat transfer between the hot and cold fluids, is estimated by the inverse method. Unlike the studies of Chen et al. [16,17], the heat flux considered in the current problem is both space and time dependent, an important feature with a much closer resemblance to reality. The system includes a fully developed pipe flow, a solid pipe wall and the heat flux applied on the pipe's external wall. Thus, the problem is an inverse heat conduction–convection problem. There are several approaches to solve an inverse problem. Among them, the conjugate gradient method (CGM) transforms the energy equation, the so called "direct problem", into an adjoint and a sensitivity equation and solves these three equations iteratively to minimize the estimation error [19–23]. In this article, we present the conjugate gradient method, which converges very rapidly and is not very sensitive to the measurement errors, to estimate the space and time dependent heat transfer rate on the pipe wall by using simulated temperature measurements. Subsequently, the distributions of temperature in the pipe can be determined as well. The conjugate gradient method is derived based on perturbation principles and transforms the inverse problem into the solutions of three problems, namely, the direct, the sensitivity, and the adjoint problems, which will be discussed in detail in the following sections.

2. Analysis

2.1. Direct problem

To illustrate the methodology of developing expressions for use in determining the unknown space and time dependent heat transfer rate, $q(x,t)$, on the external wall of a fully developed pipe flow, the following transient heat transfer problem is considered. Fig. 1 shows the geometry of a fully developed pipe flow. The pipe is of length L, radius R and thickness E, and the system's initial temperature (including pipe and fluid) is T_0. Assume the fluid is suddenly submitted to a heating process by applying a space and time variable heat flux on the wall region $D_1 < x < D_2$, while the rest of the pipe's external wall is under adiabatic conditions. The heat is then conducted inside the solid material of the pipe towards the pipe's inner wall where the heat is transferred to the cold fluid by conjugate heat transfer. It is eventually carried downstream by the forced convection to the cold fluid flow inside the pipe. The mathematical formulation of this transient heat transfer problem, covering the solid and fluid domains, can be expressed as

$$\frac{\partial^2 T_s}{\partial r^2}+\frac{1}{r}\frac{\partial T_s}{r\partial r}+\frac{\partial^2 T_s}{\partial x^2}=\frac{1}{\alpha_s}\frac{\partial T_s}{\partial t}, \tag{1}$$

$$\frac{\partial^2 T_f}{\partial r^2}+\frac{1}{r}\frac{\partial T_f}{\partial r}+\frac{\partial^2 T_f}{\partial x^2}=u_x\frac{1}{\alpha_f}\frac{\partial T_f}{\partial x}+\frac{1}{\alpha_f}\frac{\partial T_f}{\partial t}, \tag{2}$$

$$T_s=T_f=T_0, \quad \text{at } x=0, \tag{3}$$

$$\frac{\partial T_s}{\partial x}=\frac{\partial T_f}{\partial x}=0, \quad \text{at } x=L,\ 0<r<R+E, \tag{4}$$

$$\frac{\partial T_f}{\partial r}=0, \quad \text{at } r=0,\ 0<x<L, \tag{5}$$

$$T_s=T_f, \quad \text{at } r=R,\ 0<x<L, \tag{6}$$

$$k_s\frac{\partial T_s}{\partial r}=k_f\frac{\partial T_f}{\partial r}, \quad \text{at } r=R,\ 0<x<L, \tag{7}$$

$$-k_s\frac{\partial T_s}{\partial r}=q_s, \quad \text{at } r=R+E,\ D_1<x<D_2, \tag{8}$$

$$\frac{\partial T_s}{\partial r}=0, \quad \text{at } r=R+E,\ 0<x<D_1,\ D_2<x<L, \tag{9}$$

$$T_s=T_f=T_0, \quad \text{at } t=0, \tag{10}$$

where $q(x,t)$ is the heat flux applied along $r=R+E$ and $D_1<x<D_2$, k is the thermal conductivity, while $u_x=2u_{av}[1-(r/R)^2]$ with u_{av} being the average velocity. Here, the subscripts s and f refer to the solid region and the fluid region, respectively. The direct problem considered here is concerned with determination of the medium temperature when the heat transfer rate $q(x,t)$, thermal properties and initial and boundary conditions are known.

2.2. Inverse problem

For the inverse problem, $q(x,t)$ is regarded as being unknown, while everything else in Eqs. (1)–(10) is known. In addition, temperature readings taken along $r=r_m$ and $D_1<x<D_2$ are considered available. The objective of the inverse analysis is to predict the unknown space and time dependent heat transfer rate $q(x,t)$ from knowledge of these temperature readings. Let the measured temperature at the measurement positions be denoted by $Y(x_m,r_m,t)$. Then, this inverse problem can be stated as follows: by utilizing the above mentioned measured temperature data $Y(x_m,r_m,t)$, the unknown $q(x,t)$ is to be estimated over the specified domain.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized: