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Explicit representation of the implicit Colebrook–White equation

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ABSTRACT

It is shown that the Colebrook–White equation $1/\sqrt{\lambda} = -2\lg\left[2.51/Re\sqrt{\lambda} + \epsilon/3.71D\right]$ can be solved analytically for the friction factor λ. The solution contains two infinite sums. For given Reynolds numbers Re and relative roughnesses ϵ/D, one can create an own approximation with the required accuracy by adding a finite number of summands. The computing time of both the iterative calculation and several approximations is being compared. In all cases, the approximation is much faster than the iteration. Two examples for practical applications are given.

1. Introduction

The friction factor for turbulent flows in rough pipes is needed to calculate the pressure drop. It is also needed to estimate the Nusselt number with the Gnielinski equation:

$$Nu = \frac{\frac{\lambda}{8} Re Pr}{1 + 12.7\sqrt{\frac{\lambda}{8}}\left(Pr^{2/3} - 1\right)}\left[1 + \left(\frac{D}{L}\right)^{2/3}\right] \tag{1}$$

It can be calculated with the Colebrook–White equation [1]:

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left[\frac{2.51}{Re\sqrt{\lambda}} + \frac{\epsilon}{3.71D}\right] \tag{2}$$

Here, ϵ is the average roughness height. For $\epsilon = 0$, the Colebrook–White equation becomes the Prandtl equation for the friction factor in smooth tubes:

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left[\frac{2.51}{Re\sqrt{\lambda}}\right] = 2\lg\left(Re\sqrt{\lambda}\right) - 0.8 \tag{3}$$

For Eq. (3), Goudar and Sonnad [2] already presented an explicit solution.

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Nomenclature

D	diameter (m)
L	length (m)
lg	logarithm to the base 10
ln	natural logarithm
Nu	Nusselt number (dimensionless)
Pr	Prandtl number (dimensionless)
Re	Reynolds number (dimensionless)
t_a	duration of approximation (s)
t_i	duration of iteration (s)
W	Lambert W function
$\left[x \atop y \right]$	unsigned Stirling numbers of the first kind (dimensionless)
ϵ	average roughness height (m)
λ	Darcy–Weisbach friction factor (dimensionless)
τ	dimensionless duration $\tau = t_i/t_a$ (dimensionless)

Calculating the total pressure drop by integrating the local pressure drops and during numerical simulations the iterations for λ can require long computing times due to high spatial and temporal resolutions. Therefore, numerous approximations have been developed. Brkić [3] gives an overview of 20 approximations from the literature and compares them with his own equation. Brkić [4] also presented the three approximations of Boyd [5], Barry et al. [6] and Winitzki [7] that are actually approximations of the Lambert W function. In 2013, Ćojbašić and Brkić [8] developed a very accurate approximation. With this, the friction factor can be calculated with an accuracy of less than 0.01%.

2. Analytical solution

In the following, it is shown how Eq. (2) can be rearranged analytically for the friction factor λ. The result contains two infinite sums. By truncating after a finite number, approximations for the friction factor with arbitrary precision can be created. The analytical solution can be done as follows:

$$\frac{1}{\sqrt{\lambda}} = -2\lg\left[\frac{2.51}{Re\sqrt{\lambda}} + \frac{\epsilon}{3.71D}\right] = -\frac{2}{\ln(10)}\ln\left[\frac{2.51}{Re\sqrt{\lambda}} + \frac{\epsilon}{3.71D}\right] \tag{4}$$

$$\Rightarrow -\frac{\ln(10)}{2\sqrt{\lambda}} = \ln\left[\frac{2.51}{Re\sqrt{\lambda}} + \frac{\epsilon}{3.71D}\right] \tag{5}$$

$$\text{Substitution: } \frac{2.51}{Re\sqrt{\lambda}} + \frac{\epsilon}{3.71D} = Z^* \tag{6}$$

$$\Rightarrow \sqrt{\lambda} = \frac{2.51}{Re\left(Z^* - \frac{\epsilon}{3.71D}\right)} \tag{7}$$

Inserting Eqs. (6) and (7) into Eq. (5) yields

$$\ln(Z^*) = -\frac{\ln(10)}{2}\cdot\frac{Re\left(Z^* - \frac{\epsilon}{3.71D}\right)}{2.51} \tag{8}$$

$$\Rightarrow \ln(Z^*) = \frac{Re\cdot\epsilon\ln(10)}{2\cdot2.51\cdot3.71D} - \frac{ReZ^*\ln(10)}{2\cdot2.51} \tag{9}$$

$$\text{Substitution: } \frac{Re\ln(10)}{2\cdot2.51} = Re_1^* \tag{10}$$

$$\frac{Re\cdot\epsilon\ln(10)}{2\cdot2.51\cdot3.71D} = Re_2^* \tag{11}$$

$$\Rightarrow \ln(Z^*) = Re_2^* - Re_1^*Z^* \tag{12}$$

$$\Rightarrow Z^*\frac{Re_1^*}{Re_1^*} = \exp(Re_2^* - Re_1^*Z^*) \tag{13}$$

$$\text{Substitution: } Re_1^*Z^* = Z \tag{14}$$