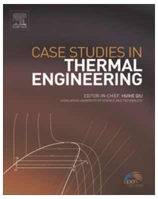
Case Studies in Thermal Engineering 5 (2015) 79–88

Contents lists available at ScienceDirect

Case Studies in Thermal Engineering

journal homepage: www.elsevier.com/locate/csite



A new method for non-Fourier thermal response in a single layer skin tissue

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A R T I C L E I N F O

Article history:
Received 26 October 2014
Received in revised form
4 January 2015
Accepted 2 February 2015
Available online 3 February 2015

Keywords:
Analytical solution
Laplace Transform
Non-Fourier
Pennes equation
Skin tissue
Thermal wave

A B S T R A C T

The non-Fourier and Fourier thermal responses in one-dimensional single layer skin tissue under selective boundary conditions are investigated by applying the Laplace Transformation method (LTM). The present method accurately describes the deviation of the temperature response of the non-Fourier model from the Fourier model and roles of important physiological parameters. A systematic exact analytical study on the discrepancies between the thermal wave model and the Pennes' bioheat model shows that the present method is an alternate reliable technique to describe the complicated bioheat problems under different boundary conditions. Three cases, namely constant skin temperature, constant and variable heat flux conditions at the skin surface have been taken to determine the tissue temperature whereas an insulated condition at the core has always been satisfied. From the result, it can be highlighted that cosine heat flux at the skin surface amplifies non-Fourier's response of temperature as well.

1. Introduction

Due to the inherent mathematical difficulties associated with biological heat transfer problems, obtaining exact solutions to them is an important thrust area. As skin plays a significant role due to its 'interfacing' between the outside materials and human inside body, analytical methods like exact Laplace transformation to solve time-dependent problems on skin bioheat transfer finds wide importance in military and space research dealing with extreme environmental conditions.

The parabolic natured bioheat transfer equation was first introduced by Pennes in 1948 [1]. Arkin et al. [2] argued that the Pennes' interpretation of the vascular contribution to heat transfer in perfused tissues fails to account for the actual thermal equilibration process between the flowing blood and the surrounding tissue. Wulff [3] also argued the same based on physical arguments and numerical results.

Based on finite propagation of thermal wave, Cattaneo [4] and Vernott [5] independently proposed hyperbolic nature conduction. This hyperbolic or phase-lag behaviour in the thermal wave may be justified if the layer has very small thickness or the time scale of the problem is very short [6].

Literature review suggests that most of the earlier analytical works were carried out on parabolic type bioheat problems. Like, Shih et al. [7] analytically studied the parabolic Pennes' bioheat equation subjected to an oscillatory heat flux boundary condition at the skin by employing Laplace transformation method. Liu [8] derived an analytical solution to the Pennes' bioheat transfer equation in three-dimensional geometry with practical hyperthermia boundary conditions and random

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http://dx.doi.org/10.1016/j.csite.2015.02.001
2214-157X/

Nomenclature

C	thermal wave speed in the medium (m s^{-1})
c_b	specific heat of blood (J kg^{-1} °C^{-1})
c_t	specific heat of tissue (J kg^{-1} °C^{-1})
k	thermal conductivity of tissue (W m^{-1} °C^{-1})
L	skin thickness (m)
m	parameter defined in Eq. (12) (m^{-1})
M	parameter defined in Eq. (13) (°C m^{-2})
n	positive integer number
$\bar{q}$	heat flux vector (W m^{-2})
q_0	surface heat flux applied to skin surface for a constant heating (W m^{-2})
q_c	constant surface heat flux, see Eq. (23) (W m^{-2})
q_{ext}	heat generated (W m^{-3})
q_m	heat flux parameter, q_w/k (m^{-2} °C)
q_{met}	metabolic heat generation (W m^{-3})
q_w	amplitude of transient heat flux (W m^{-2})
Res	residue at pole
s	Laplace parameter (s)
$S1_n$, $S2_n$	poles
S_p	notation defined in Eqs. (30), (33) and (36)
t	time (s)
T	tissue temperature (°C)
T_a	blood temperature (°C)
$T_i(x,\ 0)$	initial tissue temperature (°C)
T_s	constant skin surface temperature (°C)
T_{si}	initial skin surface temperature (°C)
W_b	notation used in Eq. (9) (s^{-1})
x	distance perpendicular to the surface (m)

Greek Symbols

α	thermal diffusivity (m^2 s^{-1})
β	notation defined in Eq. (6) (m^{-1})
γ_1, γ_2	notations defined in Eqs. (7) and (8), respectively (m^{-2} s^{-1}, m^{-2})
λ_n	Eigen value
ρ_b	density of blood (kg m^{-3})
ρ_t	density of skin tissue (kg m^{-3})
τ	thermal relaxation time (s)
ω_b	perfusion rate of blood (m^3 kg^{-1} s^{-1})
ω	frequency of surface heating (s^{-1})
θ	elevation temperature with respect to steady, $T - T_i$ (°C)
θ_0	initial elevation temperature (°C)
$\tilde{\theta}$	temperature in Laplace domain (°C)
ψ	heat addition parameter defined in Eqs. (13) and (16)

heating. Durkee et. al. [9] presented exact solutions to the classical unsteady Pennes' bioheat equation in one-dimensional multi-regional Cartesian and spherical geometries with the constant physiological parameters. In their paper, Mitra et al. [10] made experimental study on processed meat with different boundary conditions and observed wave-like phenomena in conduction heat transfer and demonstrated that the hyperbolic heat conduction model is an accurate representation, on a macroscopic level, of the heat conduction process in such biological material. A good account of numerical works in the relevant field is also reported. Xu et al. [11] reviewed previous researches and obtained numerical solutions for multi layer skin model using finite difference method. They observed large discrepancies among the predictions of the two bioheat models. Liu [12] discussed the non-Fourier heat transfer in a multi-layer skin tissue through the thermal wave model with the skin surface subjected to a step, pulse, linear, exponential and an oscillatory heating by the finite difference method. Ozen et. al. [13] studied the temperature variations in the skin when exposed to microwave for the thermal wave model of bio-heat transfer by employing finite difference method. Most of the non-Fourier bioheat problems were solved numerically and thus necessitates the need for exact solutions.

For the first time, Liu et al. [14] solved the thermal wave model of bioheat transfer (TWMBT) in a finite medium using separation of variables and the results showed distinctive wave behaviours of bioheat transfer in skin subjected to instantaneous heating. Fazlali and Ahmadikia [15] studied the non-Fourier skin-bioheat transfer under arbitrary periodic surface temperature at skin surface with the assumption of initially constant temperature throughout the domain but practically this assumption needs modification.

Liu [16] investigated the non-Fourier thermal behaviour in a living tissue by employing a modified discretization scheme based on numerical inversion of Laplace transform and expressed the findings in terms of an elevated temperature, but a representation in terms of dimensional temperature always gives a better appreciation of temperature response. A comprehensive study on flow and heat transfer of nanofluid has been investigated by many researchers [17–21].

In the present work, the accuracy of the Laplace Transformation technique is studied on a single layer model having finite thickness for the exact analysis of heat transfer in skin, where the skin is treated as a homogeneous medium with uniform, isotropic properties. A survey of the literatures suggests that most works related to non-Fourier bioheat transfer have been carried out by numerical analysis. Here, analytical solutions are obtained for the temperature response in the skin layer for both Fourier and non-Fourier heat transfer mechanisms where three boundary conditions; namely constant temperature surface heating (Case 1), constant heat flux (Case 2) and transient heat flux (Case 3) applied at skin surface are discussed. For a better practical representation, an initial temperature distribution in the domain is taken into account which is not considered in most of the relevant previous works. It is shown that the present technique is an alternate reliable analytical method to accurately describe the complicated non-linear hyperbolic bioheat problems by having a comparative study on the discrepancies between the thermal wave model and the Pennes' model. This difference is significant at initial times and