Case Studies in Thermal Engineering 3 (2014) 43–50

Contents lists available at ScienceDirect

Case Studies in Thermal Engineering

journal homepage: www.elsevier.com/locate/csite

A finite-difference method of high-order accuracy for the solution of transient nonlinear diffusive–convective problem in three dimensions

Marco Donisete de Campos [a,b,*], Estaner Claro Romão [c], Luiz Felipe Mendes de Moura [b]

[a] Federal University of Mato Grosso, Institute of Exact and Earth Sciences, Av. Governador Jaime Campos, 6390, 78600-000 Barra do Garças, Mato Grosso, Brazil
[b] Mechanical Engineering Faculty, State University of Campinas, PO Box 6122, 13083-970 Campinas, São Paulo, Brazil
[c] Department of Basic and Environmental Sciences, EEL-USP, University of São Paulo, 12602-810 Lorena, São Paulo, Brazil

ARTICLE INFO

Article history:
Received 18 November 2013
Received in revised form
13 February 2014
Accepted 7 March 2014
Available online 3 April 2014

Keywords:
Linearization technique
Nonlinear problems
Finite differences

ABSTRACT

This paper presents an efficient technique of linearization of the nonlinear convective terms present in engineering problems involving heat and mass transfer and fluid mechanics. From two numerical applications, this technique with a method of high-order finite differences is validated by numerical solution of transient nonlinear diffusive–convective problem in three dimensions.

1. Introduction

Numerical simulation of engineering problems has been successfully performed since decades ago. Problems governed by linear differential equations have been solved easily and successfully by most numerical methods proposed in the literature, among them we can mention the works involving, e.g., finite difference method, approximate methods for fractional ordinary differential equations, finite element method, spectral method and matrix approach.

Based on the high-order finite difference method, that is the focus of the work, several authors have proposed various methodologies for numerically solving the transient diffusive–convective problems. Among these papers [1], established an exponential high-order compact alternating direction implicit method for the numerical solution of unsteady 2D convection–diffusion problems using the Crank–Nicolson scheme for the time discretization and an exponential fourth-order compact difference formula for the spatial discretization. The unconditionally stable character of the scheme has been verified by a discrete Fourier analysis and the computational results showed the accuracy, efficiency and robustness of the method [2] developed a high order compact alternating direction implicit method for solving two-dimensional unsteady convection–diffusion problems. The method is unconditionally stable and second-order accurate in time and fourth-order accurate in space. Three numerical studies are carried out to demonstrate its high accuracy and efficiency and, although it

* Corresponding author at: Federal University of Mato Grosso, Institute of Exact and Earth Sciences, Av. Governador Jaime Campos, 6390, 78600-000 Barra do Garças, Mato Grosso, Brazil. Tel.: +55 663 405 317.
E-mail address: marco_donisete@yahoo.com.br (M.D. de Campos).

http://dx.doi.org/10.1016/j.csite.2014.03.001
2214-157X/

has been proposed for convection–diffusion problems, can also be used to solve pure diffusion or pure convection problems [3] using the compact difference scheme with the operator-splitting technique solved the multidimensional time fractional diffusion problem, investigating the stability and accuracy of the scheme [4] developed an exponential high order compact alternating direction implicit method for solving three dimensional unsteady convection–diffusion equations. The method is fourth-order in space, second order in time, and proved to be unconditionally stable. Three numerical experiments are performed to demonstrate its high accuracy and efficiency and to show its superiority over the classical Douglas–Gunn alternating direction implicit scheme and the Karaa's high-order alternating direction implicit scheme [5].

In this work, the aim is to present a formulation that uses an accurate linearization technique with easy to implement and that uses only one iteration at each time step, optimizing the calculation of the velocity profile.

2. Formulation

In this work, we propose a solution, by the high-order finite difference method for the transient nonlinear convection–diffusion equation in three dimensions which is given by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+u\frac{\partial u}{\partial y}+u\frac{\partial u}{\partial z}=\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{1}$$

where $u(x,y,z,t)$ is the velocity field in the x,y,z-directions and υ is the kinematic viscosity.

For the temporal discretization, considering the equation given by Eq. (1), we use the method called α family of approximation [6], in which a weighted average of the time derivative of a dependent variable is approximated on two consecutive time steps by linear interpolation of the values of the variable at two steps

$$\frac{\partial u}{\partial t}\approx\frac{\{\tilde{u}\}^{n+1}-\{\tilde{u}\}^{n}}{\Delta t^{n+1}}=(1-\alpha)\left\{\frac{\partial u}{\partial t}\right\}^{n}+\alpha\left\{\frac{\partial u}{\partial t}\right\}^{n+1} \tag{2}$$

where $0\le\alpha\le 1$, $t\in[t^n, t^{n+1}]$, $n=0,1,2,\ldots,mt$, where mt is the number of steps in time, and $\{\ \}^n$ refers to the value of the enclosed quantity at time n and $\Delta t^{n+1}=t^{n+1}-t^n$ is the $(n+1)$th time step. Adopting $\alpha=0.5$, we obtain the well-known Crank–Nicolson method to carry out the time discretization as follows:

$$\begin{aligned}\left(\frac{u^{n+1}-u^n}{\Delta t}\right)&=0.5\left(\upsilon\frac{\partial^2 u^{n+1}}{\partial x^2}+\upsilon\frac{\partial^2 u^{n+1}}{\partial y^2}+\upsilon\frac{\partial^2 u^{n+1}}{\partial z^2}-u^{n+1}\frac{\partial u^{n+1}}{\partial x}-u^{n+1}\frac{\partial u^{n+1}}{\partial y}-u^{n+1}\frac{\partial u^{n+1}}{\partial z}\right)\\&\quad+0.5\left(\upsilon\frac{\partial^2 u^{n}}{\partial x^2}+\upsilon\frac{\partial^2 u^{n}}{\partial y^2}+\upsilon\frac{\partial^2 u^{n}}{\partial z^2}-u^{n}\frac{\partial u^{n}}{\partial x}-u^{n}\frac{\partial u^{n}}{\partial y}-u^{n}\frac{\partial u^{n}}{\partial z}\right)\end{aligned} \tag{3}$$

In order to linearize the convective terms of the previous equation will be used the technique proposed by [7] according to which for a sufficiently small time step, these terms can be expanded into *a* Taylor series after the first-derivative terms. The result is as follows:

$$u^{n+1}\frac{\partial u^{n+1}}{\partial x}\approx u^{n}\frac{\partial u^{n+1}}{\partial x}+u^{n+1}\frac{\partial u^{n}}{\partial x}-u^{n}\frac{\partial u^{n}}{\partial x}. \tag{4a}$$

$$u^{n+1}\frac{\partial u^{n+1}}{\partial y}\approx u^{n}\frac{\partial u^{n+1}}{\partial y}+u^{n+1}\frac{\partial u^{n}}{\partial y}-u^{n}\frac{\partial u^{n}}{\partial y} \tag{4b}$$

$$u^{n+1}\frac{\partial u^{n+1}}{\partial z}\approx u^{n}\frac{\partial u^{n+1}}{\partial z}+u^{n+1}\frac{\partial u^{n}}{\partial z}-u^{n}\frac{\partial u^{n}}{\partial z} \tag{4c}$$

This method is commonly known as Newton's method since it provides a quadratic convergence [8]. Note that this technique does not require an iterative linearization in each time step, making quicker the calculation of u.

And, substituting Eqs. (4a)–(4c) into the convective terms of Eq. (2), it yields

$$\begin{aligned}\left(\frac{u^{n+1}-u^n}{\Delta t}\right)&=0.5\left(\upsilon\frac{\partial^2 u^{n+1}}{\partial x^2}+\upsilon\frac{\partial^2 u^{n+1}}{\partial y^2}+\upsilon\frac{\partial^2 u^{n+1}}{\partial z^2}-u^{n}\frac{\partial u^{n+1}}{\partial x}-u^{n+1}\frac{\partial u^{n}}{\partial x}+u^{n}\frac{\partial u^{n}}{\partial x}-u^{n}\frac{\partial u^{n+1}}{\partial y}\right.\\&\quad\left.-u^{n+1}\frac{\partial u^{n}}{\partial y}+u^{n}\frac{\partial u^{n}}{\partial y}-u^{n}\frac{\partial u^{n+1}}{\partial z}-u^{n+1}\frac{\partial u^{n}}{\partial z}+u^{n}\frac{\partial u^{n}}{\partial z}\right)\\&\quad+0.5\left(\upsilon\frac{\partial^2 u^{n}}{\partial x^2}+\upsilon\frac{\partial^2 u^{n}}{\partial y^2}+\upsilon\frac{\partial^2 u^{n}}{\partial z^2}-u^{n}\frac{\partial u^{n}}{\partial x}-u^{n}\frac{\partial u^{n}}{\partial y}-u^{n}\frac{\partial u^{n}}{\partial z}\right)\\&\quad\left.+u^{n+1}\frac{\partial u^{n}}{\partial x}+u^{n+1}\frac{\partial u^{n}}{\partial y}+u^{n}\frac{\partial u^{n+1}}{\partial z}+u^{n+1}\frac{\partial u^{n}}{\partial z}\right)+\frac{u^{n+1}}{\Delta t}=F\end{aligned} \tag{5}$$

where

$$F=\frac{u^n}{\Delta t}+0.5\left(\upsilon\frac{\partial^2 u^{n}}{\partial x^2}+\upsilon\frac{\partial^2 u^{n}}{\partial y^2}+\upsilon\frac{\partial^2 u^{n}}{\partial z^2}\right).$$