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Probabilistic inference of fatigue damage propagation with limited and partial information

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Abstract A general method of probabilistic fatigue damage prognostics using limited and partial information is developed. Limited and partial information refers to measurable data that are not enough or cannot directly be used to statistically identify model parameter using traditional regression analysis. In the proposed method, the prior probability distribution of model parameters is derived based on the principle of maximum entropy (MaxEnt) using the limited and partial information as constraints. The posterior distribution is formulated using the principle of maximum relative entropy (MRE) to perform probability updating when new information is available and reduces uncertainty in prognosis results. It is shown that the posterior distribution is equivalent to a Bayesian posterior when the new information used for updating is point measurements. A numerical quadrature interpolating method is used to calculate the asymptotic approximation for the prior distribution. Once the prior is obtained, subsequent measurement data are used to perform updating using Markov chain Monte Carlo (MCMC) simulations. Fatigue crack prognosis problems with experimental data are presented for demonstration and validation.

1. Introduction

Fatigue crack damage of materials exhibits significant uncertainties due to the unstable and stochastic nature of crack propagation mechanism. Accurate deterministic fatigue damage prognosis is difficult to achieve under realistic service conditions. Therefore uncertainty quantification for fatigue damage prognosis using probabilistic methods is usually required to obtain reliable results. Uncertainties in fatigue damage prognostics arise from several sources such as material properties, loading, environmental conditions and the geometry of the cracked-component. Stress intensity factor (SIF)-driven methods are commonly used to model the fatigue crack propagation rate. For example, the classical Paris' equation and its variants.[1–3] To effectively use those models for fatigue crack prognosis, sufficient fatigue testing data are required to identify model parameters. Model prediction may be unreliable when usage condition is very different from the one under

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which model parameters are calibrated[4], and updating becomes highly useful and necessary. Probabilistic fatigue damage prognostics using Bayes' rule requires a prior probability density function (PDF) of model parameters identified from a large set of experimental data[5–7] and point measurement data.[8,9] Whether to choose or obtain a prior PDF depends on what information is available. For statistical identification of model parameters, a large set of repeated tests under the same condition is required which is very expensive. Another approach, in the absence of any information, is to construct a homogeneous/uniform probability distribution that assigns to each region of the parameter space a probability proportional to the volume of the region, which is also called non-informative prior in a Bayesian context.[10] However, some non-informative priors cannot be normalized.[11–13] In such cases, methods based on the transformation group[14,15] or reference prior[16,17] can be adopted, but analysis of the specific problem is needed. Limited or partial information, such as the mean value for a specific function involving model parameters, is sometimes available from historical data or field testing. However, there is no formal rule to utilize the partial information to obtain the prior PDF of model parameters or to perform updating in the classical Bayesian framework.

To resolve the above difficulties in probabilistic fatigue damage prognostics, two major extensions are demanded: (1) a reliable initial estimation method for model parameters, which allows for estimating the model parameter PDF in a rational manner under conditions where no enough fatigue testing data are available, and (2) a general updating rule that is capable of handling different types of measurement data. The first aspect is realistic but challenging considering the fact that exhaustively performing fatigue testing for all usage conditions, particularly for unforeseen usage conditions, is not practical. The second aspect is also demanding because not all measurable data are in the form of point measurements which can directly be incorporated for updating using Bayes' rule. The objective of this study is thus not to provide a superior method to the regression method or to argue limitations of Bayes' rule. The objective of the study, however, is to develop a method allowing one to identify the prior PDF of model parameters using limited or partial information for cases where the traditional statistical identification is difficult to apply due to the limited number of data points available for a normal regression, and to formulate an updating rule that is able to handle more versatile data from usage monitoring for uncertainty reduction in prognosis results. Therefore, the underlying assumptions made in this study are: there is no or not enough of testing data for statistical identification of model parameters using normal regression methods and partial information is available for updating. In the study, the probabilistic identification of model parameters given limited or partial information is proposed based on the principal of maximum entropy (MaxEnt), and the updating rule that is capable of handling additional information other than point measurements commonly seen in the classical Bayesian analysis is formulated based on the principal of maximum relative entropy (MRE).

The remainder of the paper is organized as follows. First, the probabilistic identification of model parameters using limited or partial information is derived to obtain the prior PDF of model parameters. To evaluate the prior PDF, a numerical quadrature interpolating method is proposed. Next, the updating rule is formulated according to the principle of MRE for probability updating given additional data such as response measures to reduce the uncertainty in prognostics. The resulting posterior PDF from updating can be evaluated either by analytical solution (if there exists one) or approximation methods such as Markov chain Monte Carlo (MCMC) simulations. Following that, a few fatigue prognosis problems with experiment data are presented to demonstrate and validate the effectiveness of the overall method.

2. Probabilistic model parameter identification with limited or partial information

Under the condition that there is no fatigue testing data available to identify the prior PDF of model parameters using the normal regression method, it is possible that sparse response measurements, such as one crack size measurement from a few cracked components being monitored, are sometimes available. It is not possible to conduct parameter estimation using regression since the number of data points is one and the number of model parameters is larger than one (e.g., two-parameter Paris' equation). The key idea is to treat the mean value of the one response measure associated with each individual target system as a mathematical expectation of the mechanism model output. The expectation value can be considered as a constraint to formulate the prior PDF using the principle of MaxEnt. Given a random variable θ and its probability distribution $p(\theta) \in \mathbf{R}^+$, the information entropy[18] of $\theta \in \Theta$ is defined as

$$H(\theta) = -\int_{\Theta} p(\theta)\ln p(\theta)\mathrm{d}\theta \tag{1}$$

The principle of MaxEnt states that the desired probability distribution is the one that maximizes the entropy subject to all constraints.[19] The usual constraints are the mathematical expectations of some functions that involve the variable θ. For example, the first and second order moments of θ, such as $E_{p(\theta)}(\theta)$ and $E_{p(\theta)}(\theta^2)$ or more general $E_{p(\theta)}(f(\theta))$ can serve as the constraints. Here $f(\cdot)$ represents a general real-valued function. The desired prior distribution $p(\theta)$ can be derived using the method of Lagrange multipliers. Given a general expectation constraint $E_{p(\theta)}(f(\theta)) = F$, the Lagrangian Λ reads

$$\Lambda = -\int_{\Theta} p(\theta)\ln p(\theta)\mathrm{d}\theta + \alpha\left(\int_{\Theta} p(\theta)\mathrm{d}\theta - 1\right) + \lambda\left(\int_{\Theta} p(\theta)f(\theta)\mathrm{d}\theta - F\right) \tag{2}$$

Maximizing Λ by $\delta\Lambda/\delta p(\theta) = 0$ to obtain

$$p(\theta) = \frac{1}{Z}\exp(\lambda f(\theta)) \tag{3}$$

where $Z = \int_{\Theta} \exp(\lambda f(\theta))\mathrm{d}\theta$ is the normalizing constant, and α and λ are Lagrange multipliers. The term λ is calculated by solving

$$\frac{\partial \ln\left(\int_{\Theta} \exp(\lambda f(\theta))\mathrm{d}\theta\right)}{\partial \lambda} = F \tag{4}$$

The solution also holds true when θ is a vector of variables and $f(\theta)$ is a set of real-valued functions. For polynomial type of functions, such as $f_k(\theta) = \sum_{i=0}^{k} a_i\theta^i$, Eq. (4) has an analytical expression when $k \leqslant 2$. Higher order moments or a more complicated form of function can only be solved by numerical methods.[20,21] As mentioned above, the mechanism model can