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A new grid deformation technology with high quality and robustness based on quaternion

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Abstract Quality and robustness of grid deformation is of the most importance in the field of aircraft design, and grid in high quality is essential for improving the precision of numerical simulation. In order to maintain the orthogonality of deformed grid, the displacement of grid points is divided into rotational and translational parts in this paper, and inverse distance weighted interpolation is used to transfer the changing location from boundary grid to the spatial grid. Moreover, the deformation of rotational part is implemented in combination with the exponential space mapping that improves the certainty and stability of quaternion interpolation. Furthermore, the new grid deformation technique named ''layering blend deformation'' is built based on the basic quaternion technique, which combines the layering arithmetic with transfinite interpolation (TFI) technique. Then the proposed technique is applied in the movement of airfoil, parametric modeling, and the deformation of complex configuration, in which the robustness of grid quality is tested. The results show that the new method has the capacity to deal with the problems with large deformation, and the ''layering blend deformation'' improves the efficiency and quality of the basic quaternion deformation method significantly.

1. Introduction

Grid deformation technology has been widely used in aerospace field, such as aircraft aerodynamic configuration design and aeroelastic analysis. Some grid deformation technologies that are commonly used include infinite interpolation method,[1,2] Delaunay[3–7] background grid direct interpolation method and the radial basis function interpolation method. In infinite interpolation method, boundary point displacement is transmitted to the whole grid domain by simple algebraic interpolation, which is simple, highly efficient, but could produce intersection and distortion due to large deformation grid with poor quality. In Delaunay background grid interpolation method, the formation of Delaunay grid in each time step is treated as a background grid, and grid disturbance due to boundary movement is implemented by the deformation of Delaunay grid, where the new grid is updated through the mapping relationship between Delaunay grid and

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computational fluid dynamics (CFD) grid. This method does not need iteration, and has high efficiency, but for the complex configuration and large movement, it may cause the background map crossing, which means it will lead to illegal grid. Radial basis function interpolation method can solve the problems with large deformation, but needs a great amount of calculation and memory.

In this paper, combined with the quaternion interpolation and inverse distance weighting method, a new grid deformation technique is built and also tested in terms of grid quality, robustness and efficiency. The results show that the proposed method is robust and can ensure grid quality.

2. Grid deformation of the quaternion

Quaternion can be regarded as a complex extension of 3D space and can also be viewed as a vector in four-dimensional space. Recently, quaternion has been widely used in the flight dynamics, computer graphics, radar and wireless communications and spacecraft attitude control.[8–12] It consists of a real part and three imaginary parts. A quaternion can be expressed as the following single form[13]:

$$\begin{aligned} \boldsymbol{Q} &= q_1 + q_2 i + q_3 j + q_4 k \\ &= s + \boldsymbol{v} \\ &= [s, \boldsymbol{v}] \\ &(q_i \in \mathbf{R}) \end{aligned} \tag{1}$$

where s is the real part, $\boldsymbol{v}$ the vector, i, j, k are the imaginary units. The Quaternion that describes rotation can be expressed as[13]

$$\begin{aligned} \boldsymbol{Q} &= q_1 + q_2 i + q_3 j + q_4 k \\ &= \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\cdot\cos\alpha\cdot i + \sin\frac{\theta}{2}\cdot\cos\beta\cdot j + \sin\frac{\theta}{2}\cdot\cos\gamma\cdot k \\ &= \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\cdot\boldsymbol{n} \end{aligned}$$

where vector $\boldsymbol{n}$ is the axis of rotation and θ the angle of rotation, α, β, γ are the angles between the vector $\boldsymbol{n}$ and the three coordinate axis respectively, as shown in Fig. 1.

This quaternion can not only reflect the direction of rotation, but also the magnitude of rotation.[8,9] So the rotation of vector $\boldsymbol{R}$ can be expressed as the following Quaternion operation:

$$\boldsymbol{R}' = \boldsymbol{QRQ}^{-1} = \boldsymbol{QRQ}^{*} \quad (\|\boldsymbol{Q}\| = 1) \tag{2}$$

where $\boldsymbol{R}$ is the initial vector and $\boldsymbol{R}'$ the vector after rotation, $\boldsymbol{Q}$ the rotation quaternion. $\boldsymbol{R}$ can be regarded as a quaternion whose real part is zero.

Interpolation method is one of the most important parts when quaternion is used to achieve grid deformation. For general vector, the technique commonly used is the linear interpolation (LERP) method:

$$\begin{aligned} &\boldsymbol{T}_{\mathrm{lerp}} = (1-a)\boldsymbol{T}_1 + a\boldsymbol{T}_2 = \mathrm{lerp}(\boldsymbol{T}_1, \boldsymbol{T}_2; a) \\ &(a \in [0,1]) \end{aligned} \tag{3}$$

where $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ are the vectors to be interpolated, and a is the interpolation weight.

Because the vector calculation is commutative, linear interpolation can be extended to interpolate with any number of vectors.

In order to describe the arc between two quaternions with constant velocity in the theory of quaternion, generally the slerp interpolation method[13] will be used, which is shown as

$$\begin{aligned} &\boldsymbol{Q}_{\mathrm{slerp}} = \frac{\sin[(1-a)\Omega]}{\sin\Omega}\boldsymbol{Q}_1 + \frac{\sin(a\Omega)}{\sin\Omega}\boldsymbol{Q}_2 = \mathrm{slerp}(\boldsymbol{Q}_1, \boldsymbol{Q}_2; a) \\ &(a \in [0,1]) \end{aligned} \tag{4}$$

where $\boldsymbol{Q}_1$ *and* $\boldsymbol{Q}_2$ are the quaternions to be interpolated, Ω is the angle between two quaternion. When executing three or more than three quaternion interpolation, computation order will lead to different interpolation results due to the non-commutativity of multiplication which is shown in Fig. 2.[13]

For example,

$$\begin{cases} \boldsymbol{Q}_{123} = \mathrm{slerp}(\mathrm{slerp}(\boldsymbol{Q}_1, \boldsymbol{Q}_2; a_1, a_2), \boldsymbol{Q}_3; a_3) \\ \boldsymbol{Q}_{231} = \mathrm{slerp}(\mathrm{slerp}(\boldsymbol{Q}_2, \boldsymbol{Q}_3; a_2, a_3), \boldsymbol{Q}_1; a_1) \\ \boldsymbol{Q}_{312} = \mathrm{slerp}(\mathrm{slerp}(\boldsymbol{Q}_3, \boldsymbol{Q}_1; a_3, a_1), \boldsymbol{Q}_2; a_2) \\ \sum_{i=1}^{3} a_i = 1 \end{cases} \tag{5}$$

where $\boldsymbol{Q}_{123} \neq \boldsymbol{Q}_{231} \neq \boldsymbol{Q}_{312}$, that is, slerp cannot be extended to interpolation for multi-quaternion.

In order to solve the problem of multi-quaternion interpolation and ensure the uniqueness of the interpolation, the exponential mapping[14] interpolation method is proposed based on Lie algebra space in the literature to eliminate the non-commutativity of quaternion multiplication.

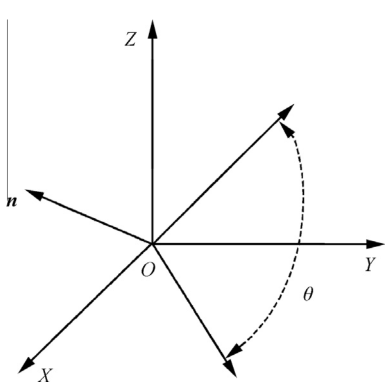

Fig. 1 A real vector rotation sketch map.

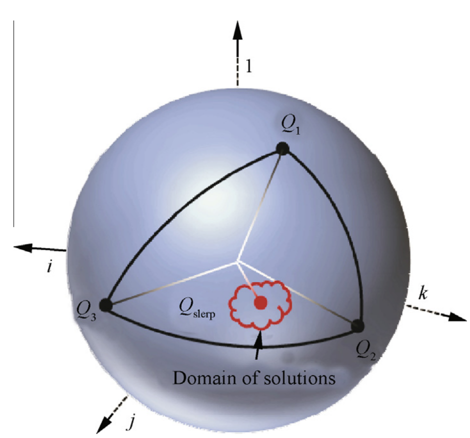

Fig. 2 Uncertainty of slerp for multi-quaternions.[13]