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Stochastic model updating using distance discrimination analysis

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Abstract This manuscript presents a stochastic model updating method, taking both uncertainties in models and variability in testing into account. The updated finite element (FE) models obtained through the proposed technique can aid in the analysis and design of structural systems. The authors developed a stochastic model updating method integrating distance discrimination analysis (DDA) and advanced Monte Carlo (MC) technique to (1) enable more efficient MC by using a response surface model, (2) calibrate parameters with an iterative test-analysis correlation based upon DDA, and (3) utilize and compare different distance functions as correlation metrics. Using DDA, the influence of distance functions on model updating results is analyzed. The proposed stochastic method makes it possible to obtain a precise model updating outcome with acceptable calculation cost. The stochastic method is demonstrated on a helicopter case study updated using both Euclidian and Mahalanobis distance metrics. It is observed that the selected distance function influences the iterative calibration process and thus, the calibration outcome, indicating that an integration of different metrics might yield improved results.

1. Introduction

Finite element (FE) models, developed to analyze engineering systems in various fields, are often used in a predictive capacity at an untested setting. The common practice of comparing model predictions to measurements at tested settings invariably yields discrepancies due to uncertainties in models and variability in testing. As a result, model updating techniques have been used to obtain models that more closely match experimental measurements.[1] There are three main sources of disagreement between FE analytical data and test measurements, all of which should be considered in model updating.

(1) Parameter uncertainty. An FE model generally involves a set of imprecise parameters of the physical structures (e.g. elastic modulus, mass density, geometric size, and spring stiffness).
(2) Modeling uncertainty. Unavoidable simplifications and idealizations (e.g. assuming a linear response, frictionless joints and the erroneous modeling of boundary

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conditions) prevent FE models from accurately representing physical characteristics.

(3) Testing variability. Due to the uncontrollable random effects of the test system, measurements are only partially reproducible.

Uncertainty is either epistemic (reducible) or aleatory (irreducible). Epistemic uncertainty is due to a lack of knowledge and consists of imprecise parameters and inexact model preform (Categories 1 and 2). For example, typical aleatory uncertainties are manufacturing tolerance and test variability (Categories 1 and 3). Regarding the treatment of epistemic uncertainty, inverse analysis techniques[2–4] are developed to update models. For the treatment of aleatory uncertainty, statistical characteristics (e.g. mean and variance) as well as intervals analysis[5,6] are typically employed.

Stochastic model updating is a procedure, in which various statistical algorithms are used to update the model to better predict measurements by considering parameter uncertainty, model form uncertainty, and test variability. Stochastic model updating includes problems such as error localization, parameter selection, feature extraction, and correlation analysis, which have been and continue to be extensively studied.[7–9]

In this paper, the authors describe a novel stochastic model updating method integrating distance discrimination analysis (DDA) and advanced Monte Carlo (MC) sampling. DDA is used here to perform test-analysis correlation (TAC), which refers to the process of determining the degree of similarity (or lack thereof) between the analytical data and measurements.[10] TAC requires a correlation metric to be defined as a function of response features from the experimental measurements and corresponding (or complementary) model predictions.[11] The DDA based test-analysis correlation procedure can provide a stochastic quantification of the disagreement based on statistical data samples.

MC, while regarded as a suitable stochastic analysis framework for the forward propagation of uncertainty due to its high precision, is demanding in regards to the computational resources necessary for its use.[12,13] Consequently, techniques such as subset simulation,[14] line sampling[15] and parallel algorithm[16] are employed to mitigate computational challenges. Although stochastic updating methods have been proposed to successfully cope with high computational resource demand, these demands have prevented applications to real life, non-trivial problems.

In this manuscript, the authors describe the use of a novel stochastic TAC procedure that integrates DDA and MC. Moreover, the authors analyze and compare the suitability of various distance functions used as correlation metrics during DDA. Furthermore, the authors enable more efficient MC by using response surface models to dramatically reduce the calculation cost. Applications in the example demonstrate the performance of the proposed approach in obtaining an updated model at an acceptable computational expense.

2. Methods of analysis

2.1. Uncertain structural system

An uncertain structural system generally includes a set of random input variables $\boldsymbol{x}$ with a nominal value $\boldsymbol{x}_0$ and variations $\Delta\boldsymbol{x}$ around this nominal value:

$$\boldsymbol{x} = \boldsymbol{x}_0 + \Delta\boldsymbol{x}. \tag{1}$$

The random input variables of interest (termed as ‘‘parameters’’) are uncertain and have significant influence on the structural response of interest (termed as ‘‘features’’). These parameters, selected based on structural characteristics and engineering judgments, are calibrated during the updating procedure. As the structures become more complex, it is not rare to encounter larger FE models involving a large amount of parameters. Parameter selection is consequently developed as a research focus to identify which parameters exhibit the most effect on the features of interest. Currently, techniques such as analysis of variance[17] and MC-based sensitively analysis[18] have been employed in this field.

Assume a set of identical test structures are constructed using identical materials and procedures but with manufacturing tolerance and material heterogeneity. A measurement sample of features can be obtained through a multi-structure multi-measurement strategy with associated test variability. The input/output random variables account for the uncertainty in both parameters and features. An uncertain structural system can be characterized as a group of complex functional relationships between p parameters and q features:

$$\begin{cases} \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{\varepsilon} \\ \boldsymbol{y} = [y_1 \; y_2 \; \dots \; y_q]^{\mathrm{T}} \\ \boldsymbol{x} = [x_1 \; x_2 \; \dots \; x_p]^{\mathrm{T}} \end{cases} \tag{2}$$

where $\boldsymbol{\varepsilon}$ is a zero mean random error.

Suppose the number of identical structures is u, and for each of the structures, α repeated measurements are executed. Consequently, the size of the measurement data sample is $m = u\alpha$, with the matrix of this sample $\boldsymbol{Y}_{\text{test}}$ assembled as

$$\begin{cases} \boldsymbol{Y}_{\text{test}} = [\boldsymbol{y}_1 \; \boldsymbol{y}_2 \; \boldsymbol{y}_3 \; \dots \; \boldsymbol{y}_q] \\ \boldsymbol{y}_j = [y_{1j} \; y_{2j} \; y_{3j} \; \dots \; y_{mj}]^{\mathrm{T}} \; (j = 1, 2, \dots, q) \end{cases} \tag{3}$$

where $\boldsymbol{Y}_{\text{test}} \in \mathbf{R}^{m \times q}$, $\boldsymbol{y}_j \in \mathbf{R}^{m \times 1}$.

For the feature sample, techniques such as moment estimation and maximum likelihood estimation can be implemented to estimate mean and variance of the data population. The mean $\bar{y}$ and variance s^2 of each of the q features are obtained from

$$\begin{cases} \bar{y}_j = \dfrac{1}{m}\displaystyle\sum_{k=1}^{m} y_{kj} = \dfrac{1}{m}\boldsymbol{y}_j^{\mathrm{T}}\boldsymbol{e}_m; \\ s_j^2 = \dfrac{1}{m-1}\displaystyle\sum_{k=1}^{m}(y_{kj} - \bar{y}_j)^2 = \dfrac{1}{m-1}(\boldsymbol{y}_j - \bar{y}_j\boldsymbol{e}_m)^{\mathrm{T}}(\boldsymbol{y}_j - \bar{y}_j\boldsymbol{e}_m) \end{cases} \tag{4}$$

where $j = 1, 2, \dots, q$; $\boldsymbol{e}_m = [11 \; \dots \; 1]^{\mathrm{T}}$; $\boldsymbol{e}_m^{\mathrm{T}}\boldsymbol{e}_m = m$. Mean vector $\bar{\boldsymbol{y}}$ and covariance matrix $\boldsymbol{C}$ of the feature sample are determined according to

$$\begin{cases} \bar{\boldsymbol{y}} = \dfrac{1}{m}[\boldsymbol{y}_1 \; \boldsymbol{y}_2 \; \dots \; \boldsymbol{y}_q]^{\mathrm{T}}\boldsymbol{e}_m = \dfrac{1}{m}\boldsymbol{Y}_{\text{test}}^{\mathrm{T}}\boldsymbol{e}_m \\ \boldsymbol{C} = \dfrac{1}{m-1}(\boldsymbol{Y}_{\text{test}} - \boldsymbol{e}_m\bar{\boldsymbol{y}}^{\mathrm{T}})^{\mathrm{T}}(\boldsymbol{Y}_{\text{test}} - \boldsymbol{e}_m\bar{\boldsymbol{y}}^{\mathrm{T}}) \end{cases} \tag{5}$$

The sample mean vector $\bar{\boldsymbol{y}} \in \mathbf{R}^{q \times 1}$ and covariance matrix $\boldsymbol{C} \in \mathbf{R}^{q \times q}$ are found to be unbiased estimators of the population mean vector $\boldsymbol{\mu}$ and the population covariance matrix $\boldsymbol{\Sigma}$.

2.2. MC simulation and response surface model

FE models of structural systems are available only with modeling simplifications and approximations. The selected