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# FrFT-CSWSF: Estimating cross-range velocities of ground moving targets using multistatic synthetic aperture radar

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**Abstract** Estimating cross-range velocity is a challenging task for space-borne synthetic aperture radar (SAR), which is important for ground moving target indication (GMTI). Because the velocity of a target is very small compared with that of the satellite, it is difficult to correctly estimate it using a conventional monostatic platform algorithm. To overcome this problem, a novel method employing multistatic SAR is presented in this letter. The proposed hybrid method, which is based on an extended space-time model (ESTIM) of the azimuth signal, has two steps: first, a set of finite impulse response (FIR) filter banks based on a fractional Fourier transform (FrFT) is used to separate multiple targets within a range gate; second, a cross-correlation spectrum weighted subspace fitting (CSWSF) algorithm is applied to each of the separated signals in order to estimate their respective parameters. As verified through computer simulation with the constellations of Cartwheel, Pendulum and Helix, this proposed time-frequency-subspace method effectively improves the estimation precision of the cross-range velocities of multiple targets.



## 1. Introduction

Space-borne multistatic synthetic aperture radar (SAR) consists of several small, cooperating satellites (a "constellation") in which electromagnetic waves transmitted by one satellite are received by the others as coherent echoes.[1] Three typical multistatic SAR constellation types are the Cartwheel, the Pendulum and the Helix, all of which can have along-track, across-track, and/or vertical baselines that are not limited to linear arrays. Multistatic SAR is more powerful than monostatic SAR in a wide range of applications including high resolution wide-swath[2] and 3/4/5-D imaging[3] as well as ground moving target indication (GMTI).[4,5] In particular, multistatic SAR is excellent at estimating the motion parameters[6] of moving targets—an important step in GMTI. One such motion parameter is velocity, which is usually represented using two orthogonal projections: range/radial velocity and cross-range velocity (also called along-track velocity). Some of the GMTI literature has focused more closely on obtaining the range/radial velocity of a moving target,[7,8] as disregarding cross-range velocity has

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no impact on extracting a moving target from its topographic background. However, this results in a defocusing of the target within the image, as there is a difference between the Doppler chirp rate of a moving target and that of static scatter. In addition, indicating the cross-range velocity with as much accuracy as the other motion parameters more fully completes the GMTI process (i.e. on an SAR image, there is no reason to indicate only the range/radial velocity while ignoring the cross-range velocity). Thus, the cross-range velocity is also computed in other GMTI work, although this is a challenging task. For example, Baumgartner and Krieger[9] obtained the cross-range velocity by searching for a matching Doppler slope that maximized the output for the target via matched filtering, while Dragosevic et al.[10] attempted to estimate the cross-range velocity by means of a fractional Fourier transform (FrFT),[11] although doing so with sufficient precision proved untenable, as the true echo is a finite length digital signal, which limits parameter resolution. Further efforts to develop conventionally effective methods using single-channel[12] and multi-channel[13,14] airborne SAR have also failed to achieve the required precision, as the speed of a space-borne platform is much higher than that of an aircraft. In light of these only partially successful efforts, it is necessary to attempt new approaches for improving precision.

Based on the suggestion of Krim and Viberg,[15] we propose array processing as one such approach. In this paper, we present a multistep procedure for developing a time-frequency-subspace method. The paper is organized as follows: First, in Section 2, we derive a novel signal form of the SAR echo for space-borne multistatic platforms in the form of an extended space-time model (ESTIM) by exploiting the space-time properties of the azimuth signal. We then in Section 3.1 design a set of FrFT-FIR filter banks for separating multiple moving targets within a range gate by utilizing the advantages of the FrFT, which has perfect time-frequency aggregation for signals with a second-order polynomial phase (such as those used in ESTIM). Next, in Section 3.2, we develop a cross-correlation-spectrum weighted subspace fitting (CSWSF) algorithm to estimate the cross-range velocities of respective separated targets. Finally in Section 4, we assess the effectiveness of the proposed method through computer simulation with the constellation of Cartwheel, Pendulum and Helix, and demonstrate that it performs better than conventional FrFT methods.

## 2. Multistatic SAR moving target formula

The scene of spaceborne multistatic SAR surveying ground moving targets is shown in Fig. 1, where the corresponding scales of satellites and moving targets are much larger than the real for description convenience. The $x$, $y$, and $z$ axes represent the along-track, cross-track, and vertical directions, respectively, and constitute a left-handed coordinate system. The multistatic SAR system described here consists of a transmitter satellite denoted as $\text{Sat}_0$ (in side-looking strip-map mode) and $N$ receiver satellites (denoted as $\text{Sat}_n$, $n = 0, 1, \ldots, N$) flying along the $x$ axis at speed $V$. At the acquisition time of the $m$th pulse $t_m = m/\text{PRF}$ (PRF is the pulse repetition frequency, $m$ is an integer, $m = 1, 2, \ldots, M$), the coordinates of the $n$th radar transceiver are $(B_{a,n} + Vt_m,\ B_{r,n},\ H + B_{v,n})$, where $H$ denotes the flight altitude of $\text{Sat}_0$, and $B_{a,n}$, $B_{r,n}$, and $B_{v,n}$ are, respectively, the along-track (also azimuth direction), cross-track (also range direction), and vertical baselines of the $n$th satellite. In particular, for the transmitter, $B_{a,0} = B_{r,0} = B_{v,0} = 0$. During the duration of the survey, there is a moving target on flat ground with velocity $\boldsymbol{v} = [v_x, v_y]$, where $v_x$ represents cross-range velocity and $v_y$ denotes range velocity, an initial location at $t_m = 0$ of $(x_0, y_0, 0)$, and a shortest range gate slant range to the transmitter of $R_0$. In Fig. 1, $\theta$ and $\varphi$ are respectively the elevation and azimuth angles of the moving target, that $\theta = \arcsin\dfrac{H}{R_0}$, $\varphi = \arctan\dfrac{y_0}{x_0}$.

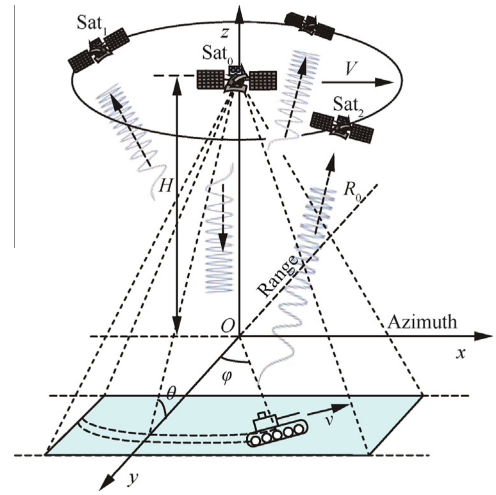


**Fig. 1** Scene of spaceborne multistatic SAR surveying ground moving targets.

### 2.1. Extended space-time model

The azimuth signal of a SAR is usually approximated with a linear frequency modulation (LFM) signal[11]; this method, however, is not optimal for estimating multistatic SAR parameters, and we will correspondingly derive the azimuth signal in an extended form. The azimuth signal of a moving target at the $n$th receiver satellite is

$$\begin{aligned} s_a(n, t_m) &= \exp\left[-\mathrm{j}\frac{4\pi\bar{R}_n(t_m)}{\lambda}\right] \\ &= \exp\left\{-\mathrm{j}\frac{2\pi}{\lambda}[r_0(t_m) + r_n(t_m)]\right\} \end{aligned} \tag{1}$$

where $r_0(t_m)$ is the range between the target and $\text{Sat}_0$, $r_n(t_m)$ is the range between the target and $\text{Sat}_n$, $\bar{R}_n(t_m) = \dfrac{1}{2}[r_0(t_m) + r_n(t_m)]$, and $\lambda$ is the wave length. The scattering coefficient is neglected for simplicity, and

$$\begin{aligned} r_n(t_m) &= \left[(x_0 + v_x t_m - B_{a,n} - Vt_m)^2 + (y_0 + v_y t_m - B_{r,n})^2 + (H + B_{v,n})^2\right]^{\frac{1}{2}} \\ &= \left[R_0^2 - 2(Vt_m - v_x t_m + B_{a,n})R_0\cos\theta\cos\varphi \right. \\ &\quad + (Vt_m - v_x t_m + B_{a,n})^2 + (v_y t_m - B_{r,n})^2 \\ &\quad \left. + 2(v_y t_m - B_{r,n})R_0\cos\theta\sin\varphi + 2B_{v,n}R_0\sin\theta + B_{v,n}^2\right]^{\frac{1}{2}} \end{aligned} \tag{2}$$