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Optimal guidance of extended trajectory shaping

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Abstract To control missile's miss distance as well as terminal impact angle, by involving the time-to-go-*n*th power in the cost function, an extended optimal guidance law against a constant maneuvering target or a stationary target is proposed using the linear quadratic optimal control theory. An extended trajectory shaping guidance (ETSG) law is then proposed under the assumption that the missile-target relative velocity is constant and the line of sight angle is small. For a lag-free ETSG system, closed-form solutions for the missile's acceleration command are derived by the method of Schwartz inequality and linear simulations are performed to verify the closed-form results. Normalized adjoint systems for miss distance and terminal impact angle error are presented independently for stationary targets and constant maneuvering targets, respectively. Detailed discussions about the terminal misses and impact angle errors induced by terminal impact angle constraint, initial heading error, seeker zero position errors and target maneuvering, are performed.

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1. Introduction

Linear optimal guidance laws with zero miss distance and terminal impact angle constraints have been extensively studied over the past several decades. As mentioned in the literature, the effectiveness of many warhead systems is closely related to the miss distance and the final impact angle. For example, to improve the attacking effect against the stiffness surface targets, the targets deep underground or the armored vehicles, a near-vertical attacking direction is often designed. For anti-radiation missiles or ballistic missiles interceptors, a certain angle impact on the targets can enhance the destruction efficiency. In addition, angle control technology is also required to enhance survivability of the missiles against increased capability of defense systems. So, to satisfy the requirements above, guidance laws considering miss distance as well as impact angle as the terminal constraints attract increasing attention in engineering practice.

The original version of the optimal guidance law with both miss distance and impact angle constraints is proposed in Ref.[1] and is further explored in Ref.[2]. In Ref.[1], the guidance law is called explicit guidance and in Ref.[2], it is called the trajectory shaping guidance (TSG) law, and they both attempt to maneuver the missile to a desired final position while controlling the final impact angle. Most of the previous literature on optimal guidance laws with impact angle constraints is based on the linear quadratic optimal control theory and the cost function is chosen as the traditional form in which the weighting

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function is a constant. Ryoo et al.[3,4] have proposed a generalized formulation of the optimal guidance law for a constant velocity missile with an arbitrary system order and studied the guidance performance for lag-free/first order autopilot. Lee et al.[5] have investigated an optimal guidance law with constraints on terminal acceleration and the final impact angle. In Ref.[6], a terminal guidance law with impact attitude angle constraints has been studied. In recent years, a new form of optimal guidance with impact angle constraints is obtained by using a new cost function that involves the integral of control energy divided by time-to-go to the nth power.[7–9] The time-to-go weighted cost function is first proposed by Kerindler[7] in 1973. He has proved that the proportional navigation guidance (PNG) with arbitrary navigation ratio $N \geqslant 3$ is also optimal if the new cost function is introduced into the conventional linear quadratic energy optimal problem. In Ref.[8], for a stationary or a slowly moving target, the new cost function above is adopted to derive the optimal guidance law with impact angle constraints and the general performance of the guidance law is investigated. Using the same cost function, Ohlmeyer et al.[8] have proposed a generalized vector explicit guidance (GENEX) law for a nonmaneuvering target. In addition, other guidance methods that control both the terminal impact position and impact angle have been proposed in Refs.[10–18]. For example, for achieving all impact angles against stationary targets or nonstationary nonmaneuvering targets in surface-to-surface engagements, a two-stage PNG law is proposed in Refs.[13, 14] by varying the PNG navigation ratio; in Ref.[18], a sliding mode-based guidance law is studied to control the terminal impact angle.

In this paper, the optimal guidance law with impact angle constraints for a constant maneuvering target or a stationary target is derived using the same cost function found[7–9] and is called the extended trajectory shaping guidance (ETSG) law. Using the Schwartz inequality,[2] closed-form solutions for the missile's acceleration command are also derived for a lag-free ETSG system. This extends the previous work on the control of terminal impact angle constraints and is the main contribution of this paper.

In the optimal guidance problems above, the time-to-go is explicitly used but is not directly measured from any devices. Ryoo et al.[4,9] have proposed an accurate and practical time-to-go calculation method taking account of the trajectory curve. In this paper, we assume that the time-to-go information is exactly known.

2. Linear quadratic optimal problem solved by the sweep method

Define the linear state equations and boundary conditions as

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \qquad (i = 1,2,\ldots,p,\ \text{where } p \leqslant m_1) \\ x_i(t_f) = \text{specified} \end{cases} \tag{1}$$

where $\boldsymbol{x}$ is m_1 dimensional state vector $(m_1 = 1,2,\ldots)$, $\dot{\boldsymbol{x}}$ is the differential of $\boldsymbol{x}$, $\boldsymbol{x}_0$ is the initial value of $\boldsymbol{x}$ at initial time t_0 and $x_i(t_f)$ is the ith value of $\boldsymbol{x}$ at terminal time t_f, $\boldsymbol{u}$ is m_2 dimensional control vector $(m_2 = 1, 2, \ldots)$, $\boldsymbol{A}$ is $m_1 \times m_1$ dimensional state matrix and $\boldsymbol{B}$ is $m_1 \times m_2$ dimensional control matrix.

The system of Eq. (1) is assumed to be fully controllable, with the control $\boldsymbol{u}$ unbounded. Considering the optimal control problem below.

Find $\boldsymbol{u}$ to minimize the cost function

$$J = \frac{1}{2}\int_{t_0}^{t_f} (\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u})\mathrm{d}t \tag{2}$$

where $\boldsymbol{Q}$ is $m_1 \times m_1$ dimensional positive semidefinite matrix and $\boldsymbol{R}$ is $m_2 \times m_2$ dimensional positive definite matrix.

The constraints Eq. (1) can be adjoined to Eq. (2) by multipliers $\boldsymbol{v}^T = [v_1, v_2, \ldots, v_p]$, then we can get

$$J = \sum_{i=1}^{p} v_i x_i(t_f) + \frac{1}{2}\int_{t_0}^{t_f} (\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u})\mathrm{d}t \tag{3}$$

where $v_i(i = 1, 2, \ldots, p)$ is the positive real multiplier of each terminal state $x_i(t_f)$.

The Euler–Lagrange equations for the optimal problem above are found to be

$$\begin{cases} \dot{\boldsymbol{\lambda}} = -\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{A}^T\boldsymbol{\lambda} \\ \boldsymbol{u} = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{\lambda} \end{cases} \tag{4}$$

In Eq. (4), $\boldsymbol{\lambda}$ is the Lagrange multiplier vector, $\dot{\boldsymbol{\lambda}}$ is the differential of $\boldsymbol{\lambda}$.

Substituting Eq. (4) into Eq. (1), we have the two-point boundary-value problem

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{\lambda}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & -\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T \\ -\boldsymbol{Q} & -\boldsymbol{A}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{\lambda} \end{bmatrix} \tag{5}$$

In Eq. (5), the initial value $\boldsymbol{x}_0$ and the terminal value $x_i(t_f)$ are the same as expressed in Eq. (1). The terminal value of $\lambda_j(t_f)$, which is the jth element of $\boldsymbol{\lambda}$ at terminal time t_f, can be rewritten as

$$\lambda_j(t_f) = \begin{cases} v_j & (j = 1, 2, \ldots, p) \\ 0 & (j = p+1, p+2, \ldots, m_1) \end{cases} \tag{6}$$

The two-point boundary-value problem above can be solved by the sweep method.[19]

Under the assumption that the specified boundary value $[x_1, x_2, \ldots, x_p]_{t=t_f}$ as linear functions of $\boldsymbol{x}$ and $[v_1, v_2, \ldots, v_p]$ as follows:

$$\boldsymbol{\psi} = \boldsymbol{U}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{v} \tag{7}$$

where $\boldsymbol{U}$ is $p \times m_1$ dimensional matrix, $\boldsymbol{G}$ is $p \times p$ dimensional matrix. $\boldsymbol{\psi}$ and $\boldsymbol{v}$ are defined as

$$\begin{cases} \boldsymbol{\psi}^T = [x_1, x_2, \ldots, x_p]_{t=t_f} \\ \boldsymbol{v}^T = [\lambda_1, \lambda_2, \ldots, \lambda_p]_{t=t_f} \end{cases} \tag{8}$$

From the linearity of Eqs. (1), (5) and (6), it is clear that $\boldsymbol{\lambda}$ is a linear function of $\boldsymbol{x}$ and $\boldsymbol{v}$, which can be expressed as

$$\boldsymbol{\lambda} = \boldsymbol{S}\boldsymbol{x} + \boldsymbol{F}\boldsymbol{v} \tag{9}$$

where $\boldsymbol{S}$ is $m_1 \times m_1$ dimensional matrix, $\boldsymbol{F}$ is $m_1 \times p$ dimensional matrix.

Since Eqs. (7)–(9) must be valid at terminal time t_f, it is clear that we have

$$\begin{cases} \boldsymbol{S}(t_f) = \boldsymbol{0} \\ U_{ji}(t_f) = F_{ij}(t_f) = \left(\dfrac{\partial \psi_j}{\partial x_i}\right)_{t=t_f} = \begin{cases} 1 & (i = j,\ i = 1, 2, \cdots, m_1) \\ 0 & (i \neq j,\ j = 1, 2, \cdots, p) \end{cases} \\ \boldsymbol{G}(t_f) = \boldsymbol{0} \end{cases} \tag{10}$$

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