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A mixed mode partition method for delaminated beam structure

Weiqiang Li [a,b], Guangxu Cheng [a], Dujin Wang [c], Jingshen Wu [b,*]

[a] *School of Chemical Engineering and Technology, Xi'an Jiaotong University, Xi'an, China*
[b] *Department of Mechanical and Aerospace Engineering & GZ HKUST Fok Ying Tung Research Institute, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong*
[c] *Beijing National Laboratory for Molecular Sciences, CAS Key Laboratory of Engineering Plastics, Institute of Chemistry, Chinese Academy of Sciences, Beijing, China*

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ABSTRACT

A delaminated beam structure of elastic material under axial forces and bending moments is analyzed. A new mode partition method is proposed to calculate the mode I and mode II stress intensity factors. Simple analytical expressions of the stress intensity factors are derived based on classical beam theories and the local continuum analysis. The physical meaning of the unknown non-dimensional parameters in the expressions, named the distribution factors, is demonstrated, which helps to better understand the relationships between the loads and the crack tip stress states. Furthermore, comparison with different mode partition methods under different conditions shows that the current method possesses the highest accuracy while retaining the advantage of simplicity.

1. Introduction

Crack propagation in delaminated beam structures is a commonly seen fracture problem. The energy release rate (ERR) and fracture mode mixity are two important parameters of this problem. For the interfacial delamination of many composite structures, the critical energy release rate strongly depends on the ratio of its shearing-mode/opening-mode components [1–3]. For path selection of a crack in brittle materials, $K_{II} = 0$ is a widely used criterion to determine crack propagation direction [4,5]. Therefore, partitioning the energy release rate into its mode I and mode II components is crucial for studying many problems. Williams conducted pioneering work on partitioning mixed mode stress intensity factors from the energy release rate [6]. However, Williams' theory was based on unjustified symmetry arguments and contained significant errors [7]. Suo and Hutchinson derived the functional forms of stress intensity factors for delamination specimens, and extracted the unknown parameters by solving the integration equation using numerical methods [5]. Bruno and Greco developed a laminated plate model incorporating transverse shear effects [8]. The interlaminar stresses were obtained by using a linear interface to model the adhesion between layers, and the energy release rate components were evaluated by using the crack tip interlaminar stresses. However, larger errors occur when the structure deviates from the even plate thickness configuration as linear shear deformable plate assumptions are adopted in the mode partition method. Bruno and Greco further improved the accuracy of mode partition by modeling the structure with mathematically subdivided beam layers [9], which in return increased the complexity of the method. Later Wang and Harvey proposed another mode partitioning method

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* Corresponding author. Tel.: +852 2358 7200.
E-mail address: mejswu@ust.hk (J. Wu).

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Nomenclature

A	crack face area, beam cross section area
b	beam width
$D_{I,M}$	normal stress distribution factor under moment loading
$D_{I,P}$	normal stress distribution factor under axial force loading
$D_{II,M}$	shear stress distribution factor under moment loading
$D_{II,P}$	normal stress distribution factor under axial force loading
E	Young's modulus
G_I	mode I energy release rate
G_{II}	mode II energy release rate
h	beam thickness
I	second moment of area of the beam
K_I	mode I stress intensity factor
K_{II}	mode II stress intensity factor
M	moment
P	axial force
r	thickness ratio between the lower and upper beams
x	distance to crack tip
σ_{yy}	normal stress along crack plane
τ_{xy}	shear stress along crack plane

based on classical beam theories [10–12]. However the partition results based on Wang and Harvey's method also have large errors, because the Euler–Bernoulli beam theory and the Timoshenko beam theory are not applicable to the crack tip region. As Schapery and Davidson have pointed out, there is insufficient information available from classical beam theories to determine the stress intensity factors [13]. Luo and Tong developed another mode partition method based on assumptions of rigid plane and average curvatures near a crack tip [14]. Their method gave accurate results for structures with a thickness ratio between the upper and lower beams close to 1, but a large error results if the thickness ratio is not close to 1. Performances of different methods will be evaluated later in this paper.

This paper aims to develop a simple mode partition method which will be accurate for all geometry configurations and all load combinations as depicted in Fig. 1. The method will be derived through classical beam theories and the non-dimensional stress distribution patterns. The unknown parameters will be extracted from the singular field at the crack tip using numerical simulations. Therefore, this method will perform better than methods which adopt assumptions and simplifications at the crack tip. This method will possess accuracy comparable to Suo and Hutchinson's method which also considered the singular field at the crack tip. As a simple, accurate method, it can be used in various applications to calculate the stress intensity factors of a DCB specimen, evaluate the mode mixity of the interfacial delamination of laminate composites, and determine the horizontal crack height of brittle materials according the $K_{II} = 0$ criterion, etc.

2. Development of the mode partition method

The delaminated beam structure is simplified to the basic plane elasticity problem as depicted in Fig. 1. Cross-sections are taken far from the crack tip and beam edges so that the stresses on the cross-sections can be described by classical beam theories. Only the axial forces P_1, P_2 and P, together with moments M_1, M_2 and M are considered and these loadings are defined as positive when acting in the directions shown in Fig. 1. The energy release rate of this system can be easily calculated through the work done by the edge loading minus the total elastic strain energy change of the structure when the crack tip propagates per unit area. Thus:

$$G = \frac{\partial(W-U)}{\partial A} = \frac{1}{2E}\left[\frac{(P_1)^2}{A_1} + \frac{(P_2)^2}{A_2} - \frac{P^2}{A} + \frac{(M_1)^2}{I_1} + \frac{(M_2)^2}{I_2} - \frac{M^2}{I}\right] \tag{1}$$

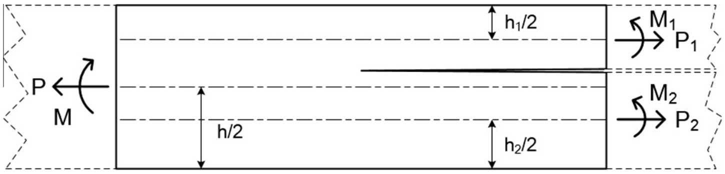

Fig. 1. Model geometry and loading conventions.