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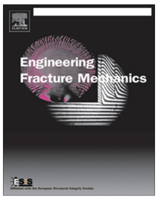

Coupled cohesive zone models for mixed-mode fracture: A comparative study

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ABSTRACT

This paper checks the consistency of some published exponential and bilinear mixed-mode cohesive zone models. The effect of coupling on traction-separation behavior and energy dissipation is investigated and the path-dependence of the debonding work of separation and failure domain is evaluated analytically and numerically. All selected models present several inconsistencies, except for the one by van den Bosch et al. (2006), which is, however, not currently formulated within a thermodynamical framework but postulated in an ad-hoc manner. We thus propose a thermodynamically consistent reformulation of this model within damage mechanics, which holds monolithically for loading, unloading, decohesion and contact.

1. Introduction

Increasing attention has been devoted in the last decades to the use of cohesive zone models (CZMs) to study mixed-mode delamination, debonding, and, more generally, crack initiation and propagation within quasi-brittle materials or at material interfaces. This is due to the computational efficiency of these models and to their versatility for numerical implementation in many areas of computational mechanics. Cohesive models describe the traction-separation behavior of interfaces before and during fracture, and are characterized by two phases, i.e. an increase of the traction up to a peak value and a subsequent decrease to zero, which describe the crack initiation and the growth of cohesive surfaces until new traction-free surfaces appear.

The basic concept of CZMs was proposed by Barenblatt [1,2] and Dugdale [3] as an alternative approach to singularity driven fracture mechanics, and has been extensively used in the literature to analyze the fracture process in a number of material systems such as concrete [4,5], polymers [6,7], ductile materials [8–10], ceramics [11], but also bimaterial systems such as polymer matrix composites [12–15] and metal matrix composites [16]. CZMs have been also used to simulate fracture under static [17–19], dynamic [11,20], and cyclic [21,22] loading conditions, delamination of layered composites at the micro- and macro-scale [23–26], delamination between a coating and a substrate [27], debonding of fiber reinforced polymer (FRP) sheets from concrete, masonry or steel substrates [28–34].

Despite the first models have been developed for single-mode fracture processes, cohesive fracture is expected to involve mixed-mode conditions, as observed in practice by experimental investigations performed on various types of lap joints [35], or interfaces between FRP sheets and flat [36,37] or curved substrates [38,39]. Mixed-mode CZMs can be classified as

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Nomenclature

g_N	normal displacement
g_T	tangential displacement
g_{Nmax}	normal displacement corresponding to the normal cohesive strength
g_{Tmax}	tangential displacement corresponding to the tangential cohesive strength
g_{Nu}	critical separation in normal direction
g_{Tu}	critical separation in tangential direction
g_m	mixed-mode relative displacement
$g_{m,u}$	ultimate value of the mixed-mode relative displacement
$g_{m,max}$	maximum value of the mixed-mode relative displacement
p_N	normal pressure
p_T	tangential pressure
p_{Nmax}	normal cohesive strength
p_{Tmax}	tangential cohesive strength
ϕ	mixed-mode fracture energy
ϕ_N	fracture energy in mode I
ϕ_T	fracture energy in mode II
W	total work of separation
W_N	normal work of separation
W_T	tangential work of separation
δW_C	contact contribution to the virtual work
q	ratio between the tangential and the normal work of separation
m	mixed-mode parameter which controls the zone of influence of mode II
r	traction-free normal separation following complete shear separation
λ	effective opening displacement
λ_P	dimensionless deformation at which the softening behavior begins
k_N	initial (elastic) stiffnesses in mode I
k_T	initial (elastic) stiffnesses in mode II
k_P	penalty contact stiffness
β	displacement-based mode-mixity
γ	material-dependent parameter for the PL criterion
η	material-dependent parameter for the BK criterion
$\mathbf{T}$	traction vector
ψ	Helmholtz energy
ψ_N^+	elastic energy associated to the normal tensile contribution
ψ_N^-	elastic energy associated to the normal compressive contribution
ψ_T	elastic energy associated to the tangential contribution
$d_i^{(j)}$	mixed-mode scalar-valued damage parameter
u	horizontal displacement
v	vertical displacement
E	elastic modulus of materials
ν	Poisson's ratio of materials
l_{pz}	length of the fracture process zone

uncoupled or coupled. Based on the former approach, the normal (i.e. mode I) traction is independent of the tangential (i.e. mode II) separation and vice versa, and a mixed-mode fracture criterion can be introduced or not. If a failure criterion is met in a point, the interface becomes unable to bear any load in that point and the local tractions drop to zero (see e.g. [35,40]). If no mixed-mode fracture criterion is defined, failure in a point is considered to occur when the energy release rates in either mode I or mode II reach their respective maximum values (see e.g. [41–44]). Uncoupled CZMs are typically used when interface separation is constrained to occur in a single predefined direction, i.e. when either mode I or mode II separation take place. Cohesive models, however, are usually applied to engineering problems where the mode of interface separation is not predefined, and thus require the use of coupled formulations. In these approaches, all the components of the traction vector depend on all the components of the interface separation.

Within the category of coupled models, cohesive models usually differ in several aspects, including the presence or not of a potential, the shape of the curve, the unloading path, and the definition of the coupling parameters. Coupled cohesive models derived from a potential and based on dimensionless coupling parameters have been, for example, proposed by Beltz and Rice [45], Needleman [46], Tvergaard [12], Tvergaard and Hutchinson [47], Xu and Needleman [18], or more recently, by Park et al. [48], McGarry et al. [49]. With this approach, however, there are some cases where the fracture energy is the same for varying mode mixities (see e.g. [47]) or where some limitations can emerge when the mode I fracture energy is different