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Computation of three-dimensional fracture parameters at interface cracks and notches by the scaled boundary finite element method

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ABSTRACT

This paper presents the computations of fracture parameters including stress intensity factors and T-stress of three-dimensional cracks and notches by the scaled boundary finite element method. The singular stress field along the crack front is approximated by a singularity at a point through a semi-analytical solution. The solution is expressed as a matrix power function which allows direct extraction of the fracture parameters based on their definitions. No singular element or asymptotic solution is required for the extraction process. The numerical examples presented which include bimaterial interface cracks and V-notches illustrate the accuracy and versatility of the proposed approach.

1. Introduction

In the framework of linear-elastic fracture mechanics (LEFM), crack propagation in brittle materials can be predicted using fracture criteria which are developed based on the work by Griffith [1] and Irwin [2]. In these criteria, the most widely used fracture parameter is the stress intensity factor (SIF) commonly denoted as K that describes the near-tip stress field. For various fracture problems, 3 modes of fracture exist. The first two modes are K_{I} and K_{II} which are commonly computed for two-dimensional (2D) in-plane problems. In 2D out-of-plane and three-dimensional (3D) problems, there is an additional mode K_{III} which is related to the out-of-plane stress mode. These SIFs are related to the first term (singular term) in the Williams' eigen series expansion [3]. The classical fracture criterion compares the computed opening mode stress intensity factor K_{I} of a cracked specimen to the critical K_{IC} (which is derived from the fracture toughness constant G_{IC}) [4]. When the critical value is exceeded, the crack will propagate in the maximum K_{I} direction. In more complex problems such as mixed mode loading, a combination of the three stress intensity factors is needed to establish accurate fracture criteria. These cases include bimaterial interfaces cracks and cracks under unsymmetrical loadings [5].

Research on SIFs has been mostly conducted for 2D models which are based on plane stress or plane strain assumptions. One of the most common numerical methods employed to compute these SIFs for 2D idealisation is the finite element method (FEM). Many different techniques are incorporated to improve the FEM accuracy to model discontinuities and interface problems. The most well-known is the 2D quarter point element which has been included in many FEM codes [6–8].

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Nomenclature

a	crack length
B	biaxiality ratio
$\mathbf{c}$	integration constants
$\mathbf{c}^{\mathrm{i}}$	integration constants related to the terms indicated by superscript i
$\mathbf{D}$	elasticity matrix
$\mathbf{E}_0$, $\mathbf{E}_1$, $\mathbf{E}_2$	coefficient matrices
G_{IC}	mode I critical fracture toughness
$\mathbf{J}(\eta,\zeta)$	Jacobian matrix on the boundary
K_{I}, K_{II}, K_{III}	stress intensity factors
K_{IC}	mode I critical stress intensity factors
$\mathbf{K}$	stiffness matrix
$\mathbf{K}(\theta,\phi)$	generalised stress intensity factors for a point at θ and ϕ
L	characteristic length
$\mathbf{L}$	differential operator matrix
$\mathbf{N}(\eta,\zeta)$	shape functions of a surface element
p	order of shape function
P_p	Legendre polynomial
$\mathbf{q}$	internal nodal forces
$\hat{r},\theta,\phi$	spherical coordinates
$\mathbf{S}$	Schur form matrix
$\mathbf{S}^{\mathrm{i}}$	Schur submatrix related to the terms indicated by superscript i
$\tilde{\mathbf{S}}^{\mathrm{s}}$	matrix of orders of singularity
T	T-stress
$\mathbf{u}$	displacement vector
$\mathbf{u}(\xi)$	nodal displacement functions on the radial lines from the scaling centre to the boundary
$\mathbf{V}$	transformation matrix
$\hat{x},\hat{y},\hat{z}$	Cartesian coordinates of an internal point
x,y,z	Cartesian coordinates of a point on the boundary
$\mathbf{Z}$	Hamiltonian coefficient matrix
$\boldsymbol{\delta u}$	crack opening displacement
$\boldsymbol{\delta u}^{\mathbf{s}}$	crack opening displacements related to singular stress terms
ϵ	oscillatory index
$\boldsymbol{\varepsilon}$	strain vector
η,ζ	scaled boundary coordinates on the boundary
η_i,ζ_i	scaled boundary coordinates of a node on the boundary
λ_i	orders of singularity
$\boldsymbol{\Phi}$	eigenvectors of matrix of orders singularity
$\boldsymbol{\Psi}^{\mathrm{i}}$	displacement modes related to the terms indicated by superscript i
$\boldsymbol{\Psi}_{\sigma}$	stress modes on a surface element
$\boldsymbol{\Psi}_{\sigma}^{\mathrm{i}}$	stress modes related to the terms indicated by superscript i
$\boldsymbol{\Psi}_{\sigma L}^{\mathrm{s}}(\theta,\phi)$	singular stress modes at a characteristic length L
$\boldsymbol{\sigma}$	stress vector
$\boldsymbol{\sigma}^{\mathrm{s}}$	singular stresses
$\boldsymbol{\sigma}^{\mathrm{T}}$	constant stresses
ξ	scaled boundary radial coordinate
$\cdot^{\mathrm{h}}$	superscript for variables related to higher-order terms
$\cdot^{\mathrm{r}}$	superscript for variables related to rigid body translational motions
$\cdot^{\mathrm{s}}$	superscript for variables related to singular stresses
$\cdot^{\mathrm{T}}$	superscript for variables related to constant stresses

Another approach for modelling stress singularities with the FEM is to utilise singular elements [9,10]. All of these improvements allow the singular stress field to be captured more accurately, hence producing more accurate SIF results. The boundary element method (BEM), which requires a fundamental solution, is another popular numerical method for the computation of SIFs e.g. through the use of path-independent J integral as shown in [11]. In BEM, the singularity can also be modelled with singular elements such as ‘traction singular quarter-point’ boundary elements proposed in [12]. In the last two decades, the extended finite element method (XFEM) has been developed to solve fracture problems e.g. to compute SIFs through the enrichment around the cracked region [13,14]. Various meshless methods such as the moving least square approximation in [15] and the element-free Galerkin method (EFGM) in [16–19] have also gained popularity to tackle some meshing difficulties in the mesh-based methods such as FEM and BEM for modelling discontinuities.