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# Micromechanics-based estimates on the macroscopic fracture toughness of micro-particulate composites

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ABSTRACT

We investigate the macroscopic fracture toughness of multi-phase materials within the framework of fracture mechanics and micromechanics. Starting with the Eshelby inclusion problem, we provide estimates on the critical energy release. We take into account the elastic and fracture properties of the micro-constituents, the microstructure and the phase volume fractions by considering three schemes: dilute, Mori–Tanaka and generalized self-consistent. In turn, the theoretical model is validated by scratch tests experiments conducted on glass-reinforced polymer composites. We also apply our theoretical framework to porous clay-based ceramics. In both cases, the agreement between experiments and theory is excellent.

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## 1. Introduction

Despite a wealth of theoretical and experimental studies, the prediction of the fracture resistance of heterogeneous materials remains elusive. While studying ellipsoidal inclusions embedded in an elasto-plastic matrix, Iizuka and Tanaka [17] found that the critical failure strain was determined by the cracking micromechanisms as well as the differences in elastic moduli between the inclusion and the surrounding medium. Taking it one step further, Raveendran et al. [27] developed upper and lower bounds for the effective fracture toughness of multi-phase materials, based on a local stress-based approach. However, they neglected the influence of the local elastic properties on the overall fracture behavior. Alternatively, Bower and Ortiz [7] considered a dispersion of tough particles into a brittle solid using a three-dimensional numerical model so as to account for the increase in toughness due to diverse mechanisms such as crack bowing, crack pinning and frictional crack bridging. Similarly, Roux et al. [29] employed a probabilistic description of the crack front to simulate the influence of the crack front waviness on the resulting fracture toughness. Both Bower and Ortiz and Roux et al. put a strong emphasis on the fracture surface geometry as opposed to the material microstructure as the driving parameter of the overall fracture resistance.

In contrast, Naito et al. [26] conducted tensile and fracture tests on ceramics-reinforced polymers nanocomposites and they showed that it is possible to enhance the fracture toughness by carefully selecting the reinforcing nanoparticle as well as the inclusion content. Carpinteri et al. [11] suggested a law of mixtures in order to upscale the fracture properties; although very appealing this is a very simplistic approach. Finally, Srivastava et al. [31] developed a theoretical

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**Nomenclature**

| | |
|---|---|
| $\mathbf{1}$ | second order unit tensor |
| $a$ | exponent corresponding to length dimension |
| $\alpha$ | constant used to assist in micromechanics derivation |
| $\alpha_i^{MT}$ | constant used to assist in Mori–Tanaka derivation for the inclusion phase |
| $\alpha_m^{MT}$ | constant used to assist in Mori–Tanaka derivation for the matrix phase |
| $\mathbb{A}$ | local strain concentration tensor |
| $\mathbb{A}_i$ | local strain concentration tensor of the inclusion phase |
| $\mathbb{A}_k^{d^{sc}}$ | local strain concentration tensor of a single inclusion for self consistent case |
| $\mathbb{A}_i^D$ | local strain concentration tensor of the inclusion phase for dilute scheme |
| $\mathbb{A}_i^{MT}$ | local strain concentration tensor of the inclusion phase for Mori–Tanaka scheme |
| $\mathbb{A}_m$ | local strain concentration tensor of the matrix phase |
| $\mathbb{A}_m^D$ | local strain concentration tensor of the matrix phase for dilute scheme |
| $\mathbb{A}_m^{MT}$ | local strain concentration tensor of matrix phase for Mori–Tanaka scheme |
| $b$ | exponent corresponding to mass dimension |
| $\beta$ | constant used to assist in micromechanics derivation |
| $\beta_i^{MT}$ | constant used to assist in Mori–Tanaka derivation for the inclusion phase |
| $\beta_m^{MT}$ | constant used to assist in Mori–Tanaka derivation for the matrix phase |
| $c$ | exponent corresponding to time dimension |
| $\mathbb{C}$ | elasticity tensor |
| $\mathbb{C}^{hom}$ | elasticity tensor for homogenized composite |
| $\mathbb{C}_i$ | elasticity tensor for the inclusion phase |
| $\mathbb{C}_k$ | elasticity tensor of a single inclusion for the self-consistent case |
| $\mathbb{C}_m$ | elasticity tensor for the matrix phase |
| $d$ | penetration depth |
| $\boldsymbol{E}$ | uniform macroscopic tensile strain |
| $E^{hom}$ | Young's modulus of the homogenized composite |
| $E_i$ | Young's modulus of the inclusion phase |
| $E_m$ | Young's modulus of the matrix phase |
| $\widetilde{\boldsymbol{E}}$ | screening strain field |
| $\varepsilon_{pot}^{hom}$ | potential energy of the homogenized composite |
| $\varepsilon_{pot}^{hom}(t^-)$ | potential energy before crack propagation of homogenized composite |
| $\varepsilon_{pot}^{hom}(t^+)$ | potential energy after complete failure of homogenized composite |
| $\epsilon$ | local strain field |
| $\epsilon_i$ | local strain in the inclusion phase |
| $\epsilon_k$ | local strain field in a single inclusion for the self consistent case |
| $\epsilon_m$ | local strain in the matrix phase |
| $F_T$ | scratch horizontal force |
| $\Gamma$ | fracture surface |
| $\mathcal{G}^{hom}$ | macroscopic energy release rate of homogenized composite |
| $\mathcal{G}_f^{hom}$ | effective fracture energy of homogenized composite |
| $\mathcal{G}_f^m$ | effective fracture energy of the matrix phase |
| $\mathbb{I}$ | fourth-order symmetric identity tensor |
| $\mathbb{J}$ | spherical portion of fourth-order symmetric identity tensor |
| $\mathbb{K}$ | deviatoric portion of fourth-order symmetric identity tensor |
| $\kappa$ | bulk modulus |
| $\kappa_{hom}$ | homogenized bulk modulus |
| $\kappa_i$ | bulk modulus of the inclusion phase |
| $\kappa_m$ | bulk modulus of the matrix |
| $K_c$ | fracture toughness |
| $K_c^{hom}$ | fracture toughness of homogenized composite |
| $K_{c_i}$ | fracture toughness of the inclusion phase |
| $K_{c_m}$ | fracture toughness of the matrix phase |
| $K_c^m$ | fracture toughness of the matrix phase |
| $K_c^\infty$ | asymptotic fracture toughness as the width-to-depth ratio approaches infinity |
| $L$ | length dimension |
| $M$ | mass dimension |