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Per unit representation of electrical magnitudes in batteries: A tool for comparison and design

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ABSTRACT

When a comparison between the performance of batteries with different characteristics, or sizing of a particular battery system in a power system (electrical grid, etc.) is carried out, the usual expression of electrical variables in terms of absolute magnitudes (Volts, etc.) has some important disadvantages derived from the wide range of values these variables can assume, as they are dependant on the "size" of the system, defined by its rated capacity, voltage or current. This makes impossible any direct comparison between different alternatives. Furthermore, it collides with the usual way power engineers use to represent and analyze the electrical power system. This paper proposes the application of a per unit system to batteries to overcome these problems. In this per unit system, all magnitudes are represented as non-dimensional values, with reference to a set of base magnitudes. Therefore, absolute values are converted into relative ones, which allow a direct comparison between different batteries. To apply a per unit system, a set of base magnitudes is studied and defined taking into account the special characteristics of a battery. The conclusion is that with a per unit system the information extracted is more accessible, direct and representative than using absolute magnitudes.

1. Introduction

The application of per unit (p.u.) systems to represent the different variables in electric power systems is an extended practice. The electric power systems, even the simplest ones, are quite complex devices; composed of many interconnected individual elements (generators, transformers, overhead lines, motors, loads, etc.) each of them defined by its own rated values. Frequently, it is necessary to compare the performance of the system in terms of voltages or currents under two different operating conditions, or the influence of marginal changes of a variable in the overall operation of the system. Or simply, it is necessary to group in a single electric circuit the parameters of the different elements which constitute the system, in order to proceed with a numeric analysis, when really these elements are connected at different voltage levels or their assigned power or current are very different. In these cases, a direct comparison between the voltage, current or power values in absolute magnitude (volts, amperes and watts, respectively) is totally irrelevant as no direct conclusion can be extracted.

Something similar happens with batteries, and in general with any energy system which includes an electrochemical device, such as fuel cells, batteries and ultra capacitors. These elements, by nature, have been a "scientific island" between two continents: the "chemical and material science" one, devoted to the study of physical phenomena and design of these devices and the "power engineering" continent, concerned with the integration of these devices and the operating characteristics of the whole system. This is perhaps the primary reason for the segregation of electrochemical devices in the use of calculus and analytic tools which have been applied to classic power systems with great success.

However, there are signs that suggest that the actual trend is just the opposite. The energetic crisis, the increasing use of renewable energy (by nature, not dispatchable) and the environmental concern have incorporated new elements and concepts, which are called to alter deeply the classical power system model, based on the centralized generation of power plants through electromagnetic devices (synchronous generators) with centralized frequency and voltage control, acting on a finite number of buses in a highly interconnected network. New paradigms, such as distributed generation, FACTS (Flexible AC Transmission Systems), energy storage devices and hydrogen as energetic vector are bound to change the established topology and operation of the future electric power systems.

To facilitate this evolution and the integration of these new elements in the conventional grid, it is necessary a change in the language in which the new elements are described. And one of these changes will surely affect the way in which the electric magnitudes of these mixed/hybrid systems are defined.

Certainly, some shy attempts have been made in this direction [1]. In batteries, the state of charge (SOC) is represented as a value relative to the rated capacity; or the discharge regime is expressed

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as, e.g., 0.2 °C for a 5 h discharge, which is a way of expressing a current as an adimensional or per unit value. But, as it will be explained later, the use of a per unit system for batteries can be applied to a greater extent and in a much more consistent and powerful way.

In this work the basic theory of a per unit system will be explained, with a special emphasis in batteries and the criteria for the selection of base or reference values. In the second place, the specific problem of batteries will be presented, explaining the need to use two different base times, in order to represent phenomena which occur in different time horizons, as it is the case of the charge/discharge processes versus the transient phenomena associated to the connection of switched electronic devices.

To this end, an equivalent circuit for a battery will be developed and its parameters will be calculated and represented as per unit values. Finally, the proposed methodology will be applied to different cases referred in the bibliography and compared with experimental results obtained for this work.

2. Basic concepts of a per unit system

A per unit system represents the electric variables as adimensional values relative to a set of reference or base values. In our electric power systems there are four basic magnitudes which describe its performance: powers, voltages, currents and impedances, which are related by (1).

$$\begin{aligned} P &= U \cdot I \\ U &= Z \cdot I \end{aligned} \tag{1}$$

As there are four variables and two equations, there are two degrees-of-freedom so the other two base values can be arbitrarily chosen. Once two base or reference values are defined (e.g. base voltage in volts and base current in amperes) the other two base values (power in watts or voltamperes and impedance in ohms) are univocally defined. For example, in a three phase system, for an apparent power of 100 MVA and 132 kV voltage, the base current and impedance are

$$\begin{aligned} I_B &= \frac{S_B}{\sqrt{3} \cdot U_B} = \frac{100 \cdot 10^6}{\sqrt{3} \cdot 132 \cdot 10^3} = 437\ \text{A} \\ \text{Z}_B &= \frac{U_B^2}{S_B} = 174.24\ \Omega \end{aligned} \tag{2}$$

In an electric system with different voltage levels coupled through transformers a unique base power is defined for the whole system, whilst the base voltage is different for each side of the transformer and usually equal to its high-side and low-side rated voltages. Therefore, different base currents and impedances will be obtained for each the voltage level.

These set of base magnitudes allow the characterization of any conventional electric system. That means, for the previous example, that if an element consumes 30 MW, the per unit power is 0.3 p.u. whatever its rated voltage, Also, a 0.85 p.u. current, converted to absolute values will be equal 0.85 p.u. multiplied by the corresponding base current, resulting in 371.45 A.

3. Definition of a set of base magnitudes for batteries

For energy storage devices such as batteries, the previously defined per unit system is inadequate for, in this case, we need to define a base value for the energy stored or released by the battery. Hence, it is essential to introduce the concept of capacity, which is related with the discharge duration according to the Peukert equation [2].

Therefore, a new set of base magnitude which includes a base capacity C_B related to a discharge time t_B must be created. Known the base capacity and discharge time, the base current can be obtained as $I_B = C_B/t_B(\text{A})$.If the "natural" choice of taking the open circuit voltage as base voltage is adopted, the rest of base values can be obtained as (3).

Table 1
Set of base magnitudes for a battery.

Base capacity	Base discharge time	Base current	Base voltage	Base power	Base impedance
C_B (A h)	t_B (h)	I_B (A)	U_B (V)	P_B (W)	Z_B (Ω)

$$\begin{aligned} P_B &= U_B \cdot I_B \\ Z_B &= \frac{U_B}{I_B} \end{aligned} \tag{3}$$

completing the set of base values necessary to describe the battery performance for stationary operation. In Table 1 all the base magnitudes are presented. Evidently, if three values are suitably selected, the rest of the base values can be calculated. However, the selection criterion of the first three base values is not arbitrary as the base values must be linearly independent. Furthermore, the base capacity, base time and base current must comply with the Peukert equation $C_B \cdot I_B^{n-1} = k_P$.

4. Per unit representation of the discharge curves

The discharge curve is an extended representation of the evolution of the battery voltage with the discharged capacity. Its frequent use makes it advisable to define it in per unit, in order to allow a direct comparison between curves obtained with different batteries.

Fig. 1a, taken from Doerffel [3] represents experimental results in which a battery apparently discharged at high discharge rate can be further discharged at a lower discharge rate, all variables being expressed in absolute values in the original work. For this case, a set of base values, presented in Table 2, have been defined for battery number one in order to allow the representation in per unit values. The absolute and per unit curves are displayed and compared in Fig. 1a and b.

Comparing the two curves in Fig. 1, Fig. 1a unveils more information than Fig. 1b, as an absolute number, e.g. 44.2 A h discharged, does not give any information about the remaining capacity without an explicit reference to the rated capacity of the battery.

On the other hand, 0.68 p.u. as the remaining capacity contains all the necessary information, as it is already referred to the rated capacity, once the set of base values are stated once and for all.

Expressing all the results as p.u. values has an added value, as it allows a simple and direct comparison with similar work carried out by different authors, with different batteries, which can have very different characteristics (rated capacity, open circuit voltage, discharge time, etc). To exemplify this, discharge test have been carried out at constant current to a VRLA Exide Tudor Maxxima battery (45 A h in 5 h, 12 V). In Table 3 a set of base values are presented for this battery and results after constant current discharge (138.42 A or 15.38 p.u.) are depicted in Fig. 2.

The comparison between Figs. 1b and 2b is easier and straightforward in p.u. values, as with a glance it is possible to known which suffers deeper discharge, or the voltage value in a particular point.

5. Per unit representation of the battery impedance

The equivalent circuit of a battery can be represented by different circuits [4–8]. In this work the circuit used is presented in