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A thermodynamic correlation between damage and fracture as applied to concrete fatigue

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A B S T R A C T

Fatigue damage in concrete is characterized by the simultaneous presence of micro and macrocracks. The theory of fracture mechanics conveniently handles the propagation of macrocracks, whereas damage mechanics precisely describes the state of microcracking. This paper provides a platform to correlate fracture mechanics and damage mechanics theories through an energy equivalence within a thermodynamic framework by equating the energy dissipated according to each theory. Through this correlation, damage corresponding to a given crack length could be obtained, and alternatively a discrete crack could be transformed into an equivalent damage zone. The results are validated using available experimental data on concrete fatigue including stiffness degradation and acoustic emission.

1. Introduction

When a material is subjected to repeated loading and unloading, it undergoes a permanent damage, even at loads well below its carrying capacity. This phenomenon more often termed as fatigue is progressive and results in a localized damage. In concrete members, fatigue damage is characterized by the formation of microcracks. With increasing number of load cycles, the microcracks coalesce to form a localized damage zone, which alternatively could be viewed as the formation of a major crack. After being subjected to further loading cycles, the major crack propagates resulting in final failure of the structural member. It is well known that in concrete, at the tip of the major crack, there exists an inelastic zone called the fracture process zone, wherein several toughening mechanisms such as microcracking, crack shielding, crack branching, and aggregate bridging, take place [1]. Thus, at a given damaged state, micro and macrocracking may be present simultaneously. Two theories that have emerged to describe failure processes in concrete are fracture mechanics and continuum damage mechanics. The former deals with the study of propagation of macrocracks in materials, whereas the latter describes the degradation of material properties due to the presence of microcracks or microvoids. In order to accurately model the phenomenon of fatigue in concrete, the two theories should be used in a unified manner. Sometimes, it is more convenient to move from one theory to another based on available parameters at hand and the ease with which the respective theory could be applied. This necessitates the correlation of the two theories based on sound physical principles so as to smoothly switch from one theory to another. Each theory has its own advantages and therefore, a flexibility to use any theory should be facilitated at the convenience of the user. For instance, a damage zone with reduced stiffness is easier to model using the finite element method compared to modeling of a crack which requires special elements. But fracture mechanics principles can be

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Nomenclature

$\mathcal{A}_k$	associated/dual variable
A	area of crack
a	crack length
B	thickness of beam
D	damage variable
d	depth of beam
E	modulus of elasticity at undamaged state
E_d	modulus of elasticity at any damaged state
f_t	tensile strength
G_F	fracture energy
l_{ch}	characteristic length
l_{dz}	length of damage zone
N	number of load cycles
N_f	number of cycles to failure
P_{min}	minimum load amplitude
P_{max}	maximum load amplitude
R	stress ratio
S	span of the beam
ΔP	load range
V_k	internal variable
V_{dz}	volume of damage zone
ΔG_I	energy release rate range in mode I
ΔY	damage strain energy release rate
Φ	dissipation potential
Φ^*	dual of dissipation potential
Φ^*_D	dual of dissipation potential in damage mechanics framework
Φ^*_F	dual of dissipation potential in fracture mechanics framework

readily used with ease in the presence of a major crack which gives a global response compared to damage mechanics which is a localized theory based on material point formulation. The two approaches can be blended so as to overcome the limitations in both of them.

The earliest attempt to use a combined damage and fracture mechanics approach, was presented by Janson and Hult [2], wherein they considered the existence of a narrow damage zone along the extended line of crack in the Dugdale's crack model. They discussed the influence of damage on tensile strength to illustrate that the actual fracture stress is less than the ideal fracture stress when damage occurs within materials. The use of the invariant J integral of fracture mechanics to describe the damage dissipative criterion of fracture in problems of crack growth was discussed by few researchers [3,4]. Pavišić and Diković [5] reported that the failure process in concrete is very complex and the use of a combined approach of three theories namely continuum damage mechanics, plasticity and fracture mechanics was suggested. Mazars and Pijaudier-Cabot [6] developed a relationship between fracture mechanics and damage mechanics theories based on thermodynamic considerations. They define an equivalent crack concept in passing from a damage zone to a fracture problem and conversely a damage zone is determined which is equivalent to a crack. Sain and Chandra Kishen [7] and Garhwal and Chandra Kishen [8] successfully applied the energy equivalence, put forward by Mazars and Pijaudier-Cabot [6] on beams subjected to fatigue kind of loading for plain concrete beams and concrete-concrete interfaces, respectively to obtain an equivalent damage zone corresponding to a crack. However, no explicit influence of fatigue was represented in their work. In this work, fracture mechanics and damage mechanics theories are correlated and explicitly applied to concrete subjected to fatigue loading within a thermodynamic framework as presented below.

In the framework of thermodynamics, dissipative phenomena are described through a dissipation potential or its dual [9]. A closed form expression for the dual of dissipation potential to describe fatigue crack propagation was derived by the authors in an already published work [11] using the concepts of dimensional analysis and self-similarity, in a fracture mechanics framework. In this work, the same is revisited with a slight modification, and following the same guidelines, a closed form expression for dual of dissipation potential for fatigue of concrete is derived in the damage mechanics framework. Fracture mechanics and damage mechanics theories are then correlated by equating the dual of dissipation potential obtained in each theory. This potential represents the energy dissipated during an irreversible process; in the present case the process represents fatigue in concrete. Through the proposed correlation, damage corresponding to a given crack length is obtained. Additionally, a discrete crack is transformed into an equivalent damage zone. This correlation is demonstrated on notched concrete beams subjected to fatigue loading under three point bending based on experimental results available in the literature. A finite element analysis is performed to compute stiffness reduction factor using both the theories by