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A modified cyclic crack propagation description

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ABSTRACT

A new form of the crack propagation law is presented for cyclic crack propagation calculations. It is characterized by a multiplicative juxtaposition of the Paris range, the threshold and critical limit and the *R*-dependence. The latter is covered by a modified Walker approach, in which the influence of small cycles at high *R*-ratios is appropriately adjusted. The concrete form of the *R*-dependence was validated by application to a wide range of materials.

1. Introduction

Ever since Paris discovered the relationship between the stress intensity factor range and the crack propagation rate for cyclic crack propagation [1], a lot of research has been devoted to the development of appropriate crack propagation laws. They mostly consist of a Paris range, denoting the linear range in a double logarithmic crack propagation rate versus stress intensity factor range diagram, and one or more modifications to cover the drop at the threshold value, the rise to infinity at the critical value and the overall *R*-dependence. Examples of such laws are the Forman law [2]

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_c - \Delta K} \tag{1}$$

(material parameters C, n, K_c) and the NASGRO law [3]

$$\frac{da}{dN} = C\left(\frac{(1-f)}{(1-R)}\Delta K\right)^n \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{\max}}{K_c}\right)^q}, \tag{2}$$

where the crack opening function f satisfies [5]

$$\begin{aligned} f &= A_0 + A_1R + A_2R^2 + A_3R^3 \quad R \geqslant 0 \\ &= A_0 + A_1R \quad -1 \leqslant R < 0 \end{aligned} \tag{3}$$

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Nomenclature

A_0	material parameter in the expression for $w_p(T)$
A_0	parameter in the NASGRO crack opening function
A_1	material parameter in the expression for $w_p(T)$
A_1	parameter in the NASGRO crack opening function
A_2	parameter in the NASGRO crack opening function
A_3	parameter in the NASGRO crack opening function
b	auxiliary parameter in the modified Walker law
B_0	material parameter in the expression for $g(T)$
B_1	material parameter in the expression for $g(T)$
C	Paris constant
$\frac{da}{dN}$	crack propagation rate
$(\frac{da}{dN})_{ref}$	reference value for the crack propagation rate
f	crack opening function in the NASGRO law
f_C	correction of the Paris law for the critical value
f_R	correction of the Paris law for the R-influence
f_{th}	correction of the Paris law for the threshold
g	material parameter in the modified Walker law
K_{max}	maximum K-value of a cycle
K_{min}	minimum K-value of a cycle
m	Paris exponent
p	Paris exponent in the Forman and the NASGRO law
p	power exponent in the NASGRO correction for the threshhold
q	power exponent in the NASGRO correction for the critical value
R	R-value = K_{min}/K_{max}
$S_{max}/2\sigma_0$	material parameter in the NASGRO crack opening function
T	temperature in Kelvin
T_{ref}	reference temperature in Kelvin
w	Walker exponent
w_n	Walker exponent for negative R-values
w_p	Walker exponent for positive R-values
α	material parameter in the NASGRO crack opening function
δ	material parameter in the correction function f_C
ΔK	stress intensity factor range
ΔK_{eff}	effective stress intensity factor range
ΔK_{th}	threshold value for the stress intensity factor range
$\Delta K_{th,intr}$	intrinsic threshold value for the stress intensity factor range, i.e. for a fully open cycle
ΔK_{ref}	material parameter in the Paris law
ϵ	material parameter in the correction function f_{th}
μ	material parameter in the expression for $w_p(T)$
ν	material parameter in the expression for $g(T)$

and

$$A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2)[\cos(\pi S_{\max}/2\sigma_0)]^{1/\alpha} \tag{4}$$

$$A_1 = (0.415 - 0.071\alpha)S_{\max}/\sigma_0 \tag{5}$$

$$A_2 = 1 - A_0 - A_1 - A_3 \tag{6}$$

$$A_3 = 2A_0 + A_1 - 1 \tag{7}$$

(material parameters: $C,\ n,\ p,\ q,\ \Delta K_{th},\ K_c,\ \alpha$ and $S_{\max}/\sigma_0$). In order to use these laws in an industrial environment, certain prerequisites must be satisfied:

- independence of the various effects,
- small number of parameters,
- accurate prediction of fatigue crack growth rates for cycles with small stress amplitudes at high R-ratios.

The first requirement originates in the fact that frequently only the Paris parameters for $R = 0$ are known for the materials at stake, the threshold value and critical value have to be estimated or taken from similar materials in the literature. If later on additional tests are performed to determine the threshold value, it is advantageous if the Paris parameters do not have to