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Coexistence of multiple attractors and crisis route to chaos in autonomous third order Duffing–Holmes type chaotic oscillators

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A B S T R A C T

We perform a systematic analysis of a system consisting of an autonomous third order Duffing–Holmes type chaotic oscillator recently introduced by Tamasevicius et al. (2009). In this type of oscillators, the symmetrical characteristics of the nonlinear component necessary for generating chaotic oscillations is synthesized by using a pair of semiconductor diodes connected in anti-parallel. Based on the Shockley diode equation and a judicious choice of state variables, we derive a smooth mathematical model (involving hyperbolic sine and cosine functions) for a better description of both the regular and chaotic dynamics of the oscillator. The bifurcation analysis shows that chaos is achieved via the classical period-doubling and symmetry restoring crisis scenarios. More interestingly, some regions of the parameter space corresponding to the coexistence of multiple attractors (e.g. coexistence of four different attractors for the same values of system parameters) are discovered. This striking phenomenon is unique and has not yet been reported previously in an electrical circuit (the universal Chua's circuit included, in spite the immense amount of related research work), and thus represents a meaningful contribution to the understanding of the behavior of nonlinear dynamical systems in general. Some PSpice simulations of the nonlinear dynamics of the oscillator are carried out to verify the theoretical analysis.

1. Introduction

Chaotic electrical circuits such as Chua circuit [1], van der Pol oscillator [2], Duffing oscillator [3], Jerk circuits [4], Colpitts oscillator [5,6], and the Hartley oscillator [7] (just to name a few) have been studied thoroughly and known to exhibit complex dynamical behaviors in the last three decades. The interest in chaotic systems lies mainly upon their complex, unpredictable behavior, and extreme sensitivity to initial conditions as well as parameter changes. Owing to the broad-band and noise like spectrum, chaotic signals are useful in various engineering applications including secure communication [8,9], image encryption [10], random bit generation [11], radar and sonar systems [12]. In a chaotic electrical circuit, the nonlinear device (e.g. semiconductor diode, bipolar junction transistor, Zener diode, nonlinear resistor) is the key element necessary for the occurrence of the chaotic behavior of the complete system. The modeling of these nonlinear devices represents the major difficulty faced

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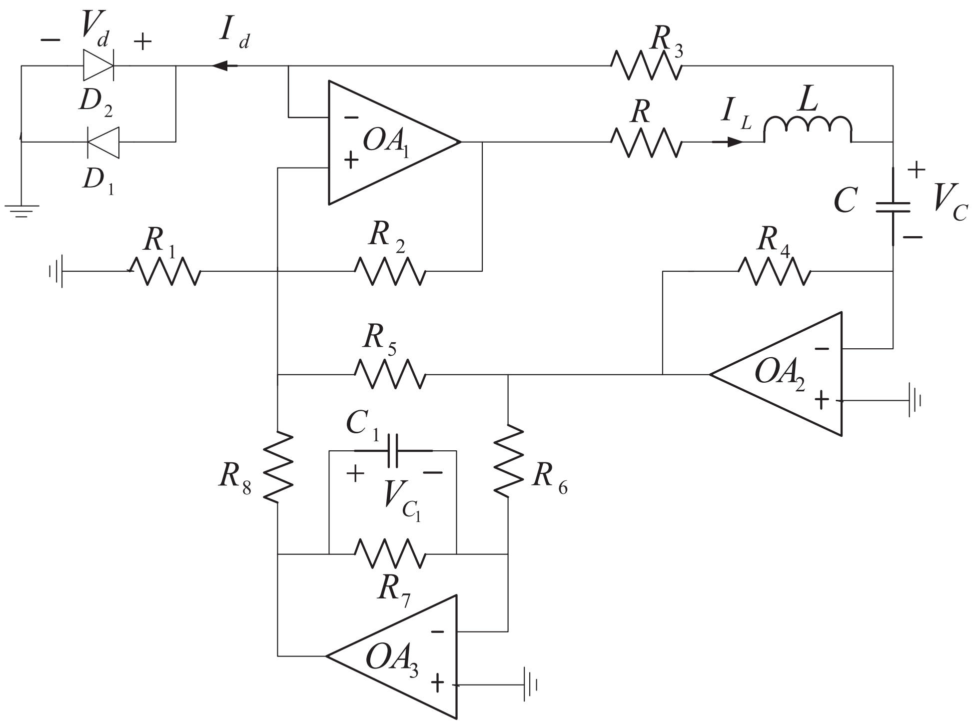

Fig. 1. Circuit diagram of the third order autonomous Duffing–Holmes chaotic oscillator [22].

during the mathematical description/representation of the whole circuit. Two approaches are commonly followed. The first approach consists of approximating the current–voltage (I–V) characteristics of the nonlinear device [5] by using piece-wise linear (PWL) functions thus leading to a PWL model of the circuit. The second approach consists of describing the characteristic of the nonlinear device by using smooth functions thus leading to a smooth (i.e. infinitely differentiable) mathematical model of the circuit [6,13–16]. A PWL model has the advantage that it can be solved analytically. However, its represents only a first order description (i.e. qualitative description) of the real circuits (e.g. cases of semiconductor diode or transistor's circuits), and consequently may lead to erroneous bifurcations compared to those observed in a real circuit. In contrast, a smooth mathematical model (e.g. based on exponential characteristics of diodes or transistors) can be exploited for a better description of both the regular and chaotic behaviors of the system. Recently, the latter approach has been considered to successfully model the dynamics of some paradigmatic chaotic/hyperchaotic oscillators such as the Colpitts oscillator [6,13], the TNC hyperchaos generator [14], the 4D Sylva Young autonomous oscillator with flat power spectrum [15], and the hyperchaotic oscillator with gyrators [16].

In this work, we consider the dynamics of an electronic analog of the autonomous Duffing type oscillator which derives from the non-autonomous Duffing–Holmes oscillator expressed by the following second order differential equation with an external periodic term:

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} - x + x^3 = a\sin(\omega t) \tag{1}$$

The electrical analog of Eq. (1), known as the Silva–Young oscillator [17], has been used to demonstrate the effects of resonant perturbations for inducing chaos [18]. Recently, the Silva–Young circuit has been essentially modified and used to test the control methods for unstable periodic orbits and unstable steady states of dynamical systems [19,20]. The modified version of the Silva–Young oscillator has been characterized both numerically and experimentally in [21]. The autonomous version of the Duffing type oscillator, introduced in literature [22], is defined by the following set of equations:

$$\begin{cases} \dfrac{d^2x}{dt^2} - b\dfrac{dx}{dt} - x + x^3 + kz = 0 \\ \dfrac{dz}{dt} = \omega_f\left(\dfrac{dx}{dt} - z\right) \end{cases} \tag{2}$$

where z stands as a third independent dynamical variable; ω_f denotes the characteristic rate; and k the feedback coefficient. We note in system (2), an opposite sign of the damping term, compared to system (1). The negative damping, $-bdx/dt$ in system (2) yields additional spirals [22]. In contrast to common approaches based on the use classical analog computers, a specific electrical