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Stationary distribution and periodic solution for stochastic predator-prey systems with nonlinear predator harvesting

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ABSTRACT

In this paper, we investigate the dynamics of the stochastic autonomous and non-autonomous predator-prey systems with nonlinear predator harvesting respectively. For the autonomous system, we first give the existence of the global positive solution. Then, in the case of persistence, we prove that there exists a unique stationary distribution and it has ergodicity by constructing a suitable Lyapunov function. The result shows that, the relatively weaker white noise will strengthen the stability of the system, but the stronger white noise will result in the extinction of one or two species. Particularly, for the non-autonomous periodic system, we show that there exists at least one nontrivial positive periodic solution according to the theory of Khasminskii. Finally, numerical simulations illustrate our theoretical results.

1. Introduction

The dynamical relationship between predators and preys has long been one of the hot topics in ecology. During the last decades, a lot of predator-prey systems have been proposed and widely used to describe the relationship between two species for food supply. For commercial purposes, the harvesting of populations is commonly used in forestry and wildlife [1,2] and the outcomes are helpful for the management of renewable resources [3,4]. Effects of harvesting on the dynamics are not only interesting in theory, but also significant for the management and sustainability of economical resources [1,2,5–7].

Economically and biologically, the harvesting for the prey species or the predator species or both is another important topic. The capture intensity mainly depends on the implemented harvesting strategy. It may range from rapid depletion to complete preservation of the population. The harvesting for the predator is sometimes more interesting than for the prey, since it is often used to control the predator size and prevent the extinction of the prey species. The harvesting function plays a key role in describing dynamic behaviors of the predator-prey system. Three types of basic harvesting functions are as follows: (i) constant harvesting, that is, the individuals are harvested with a constant number per unit of time, (ii) proportional harvesting $H(y) = hy$, which means that, the number of species harvested per unit of time is proportional to current population and (iii) nonlinear harvesting (Holling-II) $H(y) = \frac{hy}{1+by}$. Obviously, constant harvesting is random search for species and proportional harvesting is unbounded capture [9]. In some sense, nonlinear harvesting is more close to reality than the constant harvesting and proportional harvested [9,10], since $H(y) \to \frac{h}{b}$ as $y \to \infty$, which exhibits saturation effects regarding to the stock abundance and effort-level.

However, in the natural world, the populations are inevitably affected by the environmental noise. To better describe the ecological model, some authors introduced the white noise into the population systems to reveal richer and more complex dynamics, see [11–19]. Motivated by the above, we consider the predator-prey system with nonlinear predator harvesting under

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stochastic disturbance. For a stochastic Lotka–Volterra predator-prey system with regime switching, Zu et al. [20] obtained the persistence and extinction of the system in mean and gave the sufficient conditions of the existence of stationary distribution by constructing Lyapunov function under a certain condition. In addition, stationary distribution has been discussed in [12,16,21]. But in these work, the harvesting does't been taken into account. One aim of this paper is to prove the existence of a unique stationary distribution and ergodicity of the predator-prey system with nonlinear predator harvesting by constructing the suitable Lyapunov function, which does't depend on the existence and stability of the positive equilibrium.

On the other hand, due to the seasonal variation, individual lifecycle, hunting and harvesting and so on, the birth rate, the death rate of the population and other parameters will not remain constant, but exhibit a more-or-less periodicity. And yet, to our knowledge, only a few authors [22–25] investigated the existence of periodic solutions of the stochastic non-autonomous predator-prey system. In the article, we consider the existence of periodic solutions of a non-autonomous predator-prey system with stochastic disturbance.

The rest of the paper is organized as follows. In the next section, we describe the source of a predator-prey model under stochastic disturbance when the predator species is harvested with Holling-II harvesting function. Section 3 gives the existence and uniqueness of the global positive solution. In Section 4, the persistence and extinction of the predator in mean are discussed by the comparison principle. In Section 5, we show that, the autonomous system exists a unique stationary distribution and it is ergodic under a certain condition. Numerical simulations illustrate our theoretical results, shown in Section 6. For non-autonomous periodic system, the existence of nontrivial positive periodic solution is obtained in Section 7.

2. Formulation of mathematical models

Lokta [26] and Volterra [27] first proposed a Lotka–Volterra predator-prey model described by

$$\begin{aligned} \frac{dx}{dt} &= a_1x(t) - ax(t)y(t), \\ \frac{dy}{dt} &= -dy(t) + \eta ax(t)y(t), \end{aligned} \tag{2.1}$$

where $x(t)$,$y(t)$ denote the biomass densities of prey and predator at time t, respectively. a_1 and $-d$ are their intrinsic growth rates in the absence of each other. a is the prey's rate of change due to interaction. η is the conversion rate of eaten prey into new predator. In this model, the preys grow infinitely without predators, which is unreasonable when the natural resource is limit. Thus, a Logistic self-limitation term is often added to the prey equation. That is,

$$\begin{aligned} \frac{dx}{dt} &= x(t)(a_1 - b_1x(t)) - ax(t)y(t), \\ \frac{dy}{dt} &= -dy(t) + \eta ax(t)y(t), \end{aligned} \tag{2.2}$$

where $\frac{a_1}{b_1}$ is the environmental maximum carrying capacity of the prey in the absence of the predator. Turchin [8] has shown that, the system (2.2) has a unique interior equilibrium which is globally stable.

For commercial significance, the predator is continuously being harvesting and will remain permanent for a long time. Gupta et al. [10] considered the prey-predator system with nonlinear predator harvesting:

$$\begin{aligned} \frac{dx}{dt} &= x(t)(a_1 - b_1x(t)) - ax(t)y(t), \\ \frac{dy}{dt} &= -dy(t) + \eta ax(t)y(t) - \frac{hy(t)}{1+by(t)}, \end{aligned} \tag{2.3}$$

which shows that, the harvesting function exhibits a richer dynamics than the model (2.2). And the system (2.3) has always an axis equilibrium $E_1(\frac{a_1}{b_1}, 0)$, which is locally asymptotically stable if $b_1(d+h) > \eta aa_1$ and is unstable if $b_1(d+h) < \eta aa_1$. And when $b_1(d+h) < \eta aa_1$, the system (2.3) has a unique positive equilibrium $E(x^*, y^*)$, where y^* is the unique positive root of the quadratic equation:

$$\eta a^2by^2 + (dbb_1 - \eta a_1ab + \eta a^2)y + b_1(d+h) - \eta aa_1 = 0,$$

and $x^* = \frac{a_1 - ay^*}{b_1}$. By Theorem (5.4) of [10] and the minor changes, we know that $E(x^*, y^*)$ is stable if $h < \frac{(a_1-ay^*)(1+by^*)^2}{by^*}$ and unstable if $h > \frac{(a_1-ay^*)(1+by^*)^2}{by^*}$. And the system (2.3) undergoes a Hopf bifurcation when $h = \frac{(a_1-ay^*)(1+by^*)^2}{by^*}$.

In reality, the species are more or less disturbed by environment noises. We assume the intrinsic growth rates a_1 and d of the prey and the predator are disturbed with

$$a_1 \to a_1 + \alpha\dot{B}_1(t), d \to d + \beta\dot{B}_2(t),$$