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Asymmetric counter propagation of domain walls

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ABSTRACT

Far from equilibrium systems show different states and domain walls between them. These walls, depending on the type of connected equilibria, exhibit a rich spatiotemporal dynamics. Here, we investigate the asymmetrical counter propagation of domain walls in an in-plane-switching cell filled with a nematic liquid crystal. Experimentally, we characterize the shape and speed of the domain walls. Based on the molecular orientation, we infer that the counter propagative walls have different elastic deformations. These deformations are responsible of the asymmetric counter propagating fronts. Theoretically, based on symmetry arguments, we propose a simple bistable model under the influence of a nonlinear gradient, which qualitatively describes the observed dynamics.

1. Introduction

Macroscopic systems influenced by injection and dissipation of energy and/or matter typically exhibit coexistence of different stable states– - this feature is usually denominated multistability [1–4]. Inhomogeneous initial conditions caused by inherent fluctuations generate spatial domains, which are separated by interfacial domain walls. These interfaces are known as front interfaces or domain walls [3,4]. Interfaces between these metastable states appear in the form of propagating fronts and give rise to a rich spatiotemporal dynamic [5–8]. Front dynamics have been observed in several contexts such as walls separating magnetic domains [9,10], directed solidification processes [11], nematic liquid crystals [12], oscillating chemical reactions [13], and fluidized granular media [14], among others. According to the dynamical system theory, in one spatial dimension, a front is a nonlinear solution that is identified in the co-moving frame system as a heteroclinic orbit linking two spatially extended uniform states [15]. The front type solutions can be regarded as a particle-type one, i.e., they can be characterized by a set of continuous parameters such as position, core width and so forth. The front propagation depends on the nature of the states that are being connected. For example, in the case of a front connecting a stable and an unstable state, its speed is not unique but determined by the initial conditions [16]. This scenario changes for a front connecting two stable uniform states. For variational or gradient systems, the most stable state invades the other one, in order to minimize its nonequilibrium energy or Lyapunov functional, in this sense, the front is always propagating towards the higher energy state [8]. There is only one point in the parameter space for which the front is motionless. Commonly called as Maxwell's point, it is the point for which both connected states have exactly the same energy [17]; close to this point, based on variational methods, one can analytically determine the front speed. Furthermore, far from the Maxwell's point, implicit expressions for the front speed can be obtained for variational systems [8], through the solution of the corresponding nonlinear eigenvalue problem.

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In a bistable isotropic system, one expects that two counter-propagating fronts with the same speed can be created through a finite perturbation over the less favorable state, thus, making the most stable state to emerge. However, recently we have observed that perturbations of an Ising type walls in nematic liquid crystals with reflection symmetry generate two asymmetric counter-propagating fronts, each with a different speed and shape [18] [See Supplemental material for a movie that shows an example of asymmetric counter-propagating of domain walls in an inplane-switching cell filled with a nematic liquid crystal]. The perturbations are generated by the presence of glass spheres inside the liquid crystal sample. The dynamical behavior observed is common in systems under the influence of an external flow, i.e. drifting or convective systems [19]. In such case, the front that propagates in the drag force direction spreads faster than the one which propagates in the opposite direction. Likewise, the speed difference between the fronts accounts for the drag force. In addition, anisotropic propagation of point defect solutions with opposite topological charges has been reported in liquid crystals [20–22]. In this case the asymmetry of the propagation is due to the backflow around moving defects.

In this paper, we investigate in more detail the counter-propagation of asymmetrical domain walls connecting different molecular-orientation configurations in an in-plane-switching cell filled with a nematic liquid crystal without a flow, which we have recently observed [18]. These domain walls are triggered by the presence of glass bead within the sample. Experimentally, we characterize the profile and the speed of these fronts with respect to the amplitude and the frequency of the applied voltage to the liquid crystal cell. Based on the liquid crystal molecular orientation induced by the glass bead, we elucidate that the fronts generated by these spheres have different elastic deformations at the core of the fronts. These deformations are responsible for the asymmetry in the shape and speed of the fronts. Based on symmetry arguments, we propose a simple phenomenological equation – a bistable model under the influence of a nonlinear gradient – to describe the asymmetric counter-propagating fronts without flow. Analytically, we characterize the shape and the speed of the asymmetric counter-propagating fronts which qualitatively describes the observed dynamics. We evidence experimentally the kink formation due to the collision of two asymmetrical fronts and explain this phenomenon with a generalization of our simple phenomenological model.

2. Experimental front propagation

2.1. Experimental setup

To investigate the propagation of domain walls, we have considered an in-plane-switching cell filled with a nematic liquid crystal. The experimental setup under study is depicted on Fig. 1. A layer of *E*7 nematic liquid crystal is inserted between two glass plates, thickness $g = 1$ mm, with a cell gap $d = 8.8 \pm 0.2\,\mu$ m. The elastic constants of the liquid crystal under consideration are, respectively, $K_1 = 11.2$, $K_2 = 6.8$, and $K_3 = 18.6$ ($\times 10^{-12} N$). The parallel and the perpendicular dielectric constants are $\varepsilon_{\parallel} = 18.96$ and $\varepsilon_{\perp} = 5.16$ [23–26]. We consider an in-plane-switching cell, with a homogeneous planar alignment (following *y*-axis, cf. Fig. 1) and a perpendicular rubbing to the electric field (Instec, *IPS*02*A*88*uX*00). The indium tin oxide (ITO) electrode width and the gap width are the same, $e = 15\,\mu$ m. The height of the electrodes is negligible (∼25 nm) compared to the cell thickness ($d = 8.8\,\mu$ m). The active zone is a square of side $l = 1$ cm. Under these settings, we can consider the cell in a good approximation as an infinite thin film medium. The electrodes are aligned in the direction of *y*-axis, that is, the molecules are anchored parallel to the electrodes (see Fig. 1). The electrodes are connected to a function generator, applying an alternating current voltage with frequencies ∼1 Hz–10 MHz and an amplitude ∼0 Vpp–20 Vpp (Volt peak-to-peak). The cell is illuminated with a white light placed between a polarizer *P* and an analyzer *A*. In order to have a better information about the molecular orientation the polarizers can be placed parallel (//) or perpendicular ($\perp$) related to each other. The microscope magnification used is 20 × or 50 × . The liquid crystal dynamics is measured and recorded through a charge couple device camera (CCD) connected to an optical microscope.

2.2. Experimental observation of asymmetrical counter propagative fronts

We consider the system under study as an infinite liquid crystal medium. A good approximation of this, is to observe in the middle of the sample a small portion of the cell. This midplane is schematically depicted in Fig. 3d. By direct observation, without applying a voltage to the sample, only the electrode bands can be detected. Fig. 3a and top panel in Fig. 2a show these electrode bands characterized by dark zones. Note that there is a black bead between two consecutive electrodes. This bead is a glass sphere used to fix the thickness between the two glass plates, with diameter of 8.8 μ m. The glass sphere creates a 3D local perturbation in the molecules orientation around it. Applying a voltage with an amplitude of 20 Vpp and a frequency of $f = 1$ kHz, the system exhibits two asymmetric domain walls propagating towards both sides from the glass sphere, following *y*-axis (cf. Fig. 2). These fronts connect two different molecular orientations, which correspond to black bands observed over the electrodes and between them. Figs. 2 and 4 illustrate the front profiles. To demonstrate the constancy of front velocity, we represent on Fig. 2 dotted lines, blue for the left front and green for the right one, which match with the front positions for every snapshot. We note that each front propagates with constant speed, however, different between each other.

The black curves are the consequence of the molecular orientations in the sample which modify the light polarization and do not allow the light to cross the analyzer. As a result of the electrode shapes, the states that connect the fronts are not uniforms in the vertical direction. Between the gap and the middle electrode, the system exhibits three equilibria, represented by α, β and γ in Fig. 4a, and observed thanks to the use of crossed polarizers. The fronts only connect two particular molecular orientations. One is positioned in the center of the gap region (α-state, see Figs. 3c and 4 a) and the other one close to the center of the electrode