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# An improved stability characterization for aeroelastic energy harvesting applications

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ABSTRACT

An enhanced stability characterization for aeroelastic energy harvesters is introduced by using both the normal form of the Hopf bifurcation and shooting method. Considering a triangular cylinder subjected to transverse galloping oscillations and a piezoelectric transducer to convert mechanical vibrations to electrical power, it is demonstrated that the nonlinear normal form is very beneficial to characterize the type of instability near bifurcation and determine the influence of structural and/or aerodynamic nonlinearities on the performance of the harvester. It is also shown that this tool is strong in terms of designing reliable aeroelastic energy harvesters. The results show that this technique can accurately predict the harvester's response only near bifurcation, however, cannot predict the stable solutions of the harvester when subcritical Hopf bifurcation takes place. To cover these drawbacks, the shooting method is employed. It turns out that this approach is beneficial in determining the stable and unstable solutions of the system and associated turning points. The results also show that the Floquet multipliers, obtained as the by-product of this method, can be used to characterize the response's type of the harvester. Thus, the normal form of the Hopf bifurcation and shooting method predictions can supplement each other to design stable and reliable aeroelastic energy harvesters.

## 1. Introduction

To operate self-powered sensors, actuators, and other electronic devices, various types of wasted natural energy including thermal [1,2], light [3], and kinetic energy [4,5] have been used to generate inexhaustible electrical energy. Such energy harvesters have been proposed to replace small batteries that have a finite life span or would require expensive and time consuming maintenance. For powering electronic devices, mechanical energy has received significant attention in the last decade. Harvesting mechanical energy through converting vibrations to electrical energy can be achieved using either electrostatic [6], electromagnetic [7,8], magnetostrictive [9], or piezoelectric [5,10,11] transduction mechanisms. Of these mechanisms, the piezoelectric option has received the most attention because of its ease of application, non-reliance on input voltage, it can effectively be placed in small volumes, and it can harvest energy over a wide range of frequencies.

For the purpose of piezoelectric energy harvesting, two major ambient excitation categories have been considered which are base [5,10,11] and flow-induced [12–17] vibrations. To date, most of the proposed energy harvesting devices from mechanical

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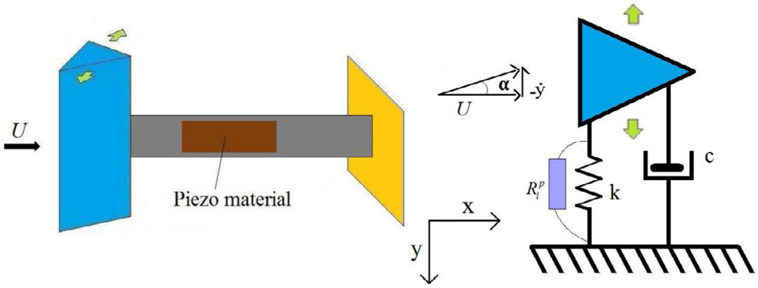

**Fig. 1.** Schematics of a galloping-based piezoelectric energy harvester (left) cantilever-based and (right) spring-mass-damper.

vibrations have concentrated on exploiting base vibrations. More recently, several research studies have focused on the concept of energy harvesting from aeroelastic or flow-induced vibrations, such as flutter of airfoil sections [18–21], vortex-induced vibration (VIV) of circular cylinders [13,16,22], galloping of prismatic structures [15,23–25], and wake galloping of parallel cylinders [26,27].

Bryant and Garcia [19] showed, both theoretically and experimentally, that electrical power can be generated from aeroelastic vibrations using an airfoil section attached to a cantilever beam. Erturk et al. [12] studied experimentally the concept of energy harvesting from a two-degree of freedom airfoil section near flutter boundary. Abdelkefi et al. [21] focused on the nonlinear aspects of harvesting energy from airfoil sections. Akaydin et al. [13] and Dai et al. [16] investigated the concept of energy harvesting from VIV of a circular cylinder attached to a bimorph piezoelectric cantilever beam. They demonstrated that maximum levels of the harvested power are obtained when the shedding frequency matches the fundamental natural frequency of the harvester.

Due to its design simplicity compared to the flutter- and VIV-based energy harvesters, recent research studies have focused on the concept of energy harvesting from transverse galloping oscillations of prismatic structures. This phenomenon occurs when the wind speed exceeds a critical value and hence the bluff body starts to oscillate [28]. Sirohi and Mahadik [23] and Abdelkefi et al. [24] studied the concept of energy harvesting from galloping of prismatic structure that has an equilateral triangle section. They showed that their harvester can generate enough power to operate most of the commercially available wireless sensors. The effects of the cross-section geometry and ambient temperature on the onset speed of galloping and the level of the harvested power have been investigated in many research studies [25,29–31].

It is well-known that the galloping phenomenon is accompanied by a Hopf bifurcation which can be supercritical or subcritical. In the supercritical case, the harvester has only stable solutions for wind speed values larger than the onset speed of galloping. On the other hand, unstable solutions are present in the subcritical cases for wind speed values smaller than the onset speed of galloping. The presence of these unstable solutions are dependent on the initial condition (IC) values. To characterize the type of instability and determine the unstable solutions, two different approaches are used, namely, the normal form of the Hopf bifurcation and the Floquet multipliers obtained from the shooting method and used for the construction of periodic solutions. In this work, structural and aerodynamic nonlinearities are present. The aerodynamic nonlinearity is associated to the galloping force and is defined by the flow parameters and the properties of the prismatic structure. As for the structural nonlinearity, hardening or softening springs can be used. The rest of this paper is organized as follows: In Section 2, a lumped-parameter model representing the dynamics of the harvester is presented. In Section 3, the normal form of the Hopf bifurcation is presented to characterize the type of instability and determine the contributions of the aerodynamic and structural nonlinearities on the performance of the harvester. In addition, the shooting method is used to construct the periodic solutions and determine the Floquet multipliers to characterize their stability.

## 2. Electro-aeroelastic model formulation

A galloping-based aeroelastic energy harvester is considered. This harvester consists of a tip mass isosceles triangular section ($\delta = 30^\circ$) attached to a multilayered cantilever beam, as shown in Fig. 1(a). The multilayered beam is composed of substrate and piezoelectric layers. The piezoelectric layer is bonded by two-in-plane electrodes of negligible thicknesses connected to an electrical load resistance $R$. When subjected to an incoming flow, this harvester can undergo galloping oscillations in the transverse direction. The Den Hartog stability criterion [28] states that a section of a bluff body on a flexible support is susceptible to galloping when the linear term associated with the velocity of the aerodynamic galloping force is positive. For the harvester under investigation, the onset speed of galloping is determined when the electromechanical damping of the harvester changes sign from a positive to a negative value due to the aerodynamic galloping force. The nonlinear term of the galloping force affects the amplitude of the ensuing limit-cycle oscillations and hence the level of the harvested power.

Because the main focus in this study is on developing an enhanced stability methodology for aeroelastic energy harvesting systems, a lumped-parameter model is used to express the governing equations of the coupled electromechanical system