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Bouncing and dynamic trapping of a bistable curved micro beam actuated by a suddenly applied electrostatic force

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ABSTRACT

In this work, the results of numerical investigations of the transient dynamics of a stress-free initially curved bistable double clamped micro beam actuated by a suddenly applied electrostatic force are presented. The analysis is based on a reduced order (RO) model derived through the Galerkin decomposition. Two beam configurations and two corresponding loading scenarios are considered. In the first case, the beam, which manifests two stable equilibria both accessible under quasi-static loading, is subjected to a suddenly applied (step function) voltage. Under such a signal, the beam may snap into the second stable configuration or bounce back to its initial position. We map the regions of the various types of response on the actuation voltage – quality factor plane. In the second case, the configuration of the beam is such that the second equilibrium is inaccessible neither under quasi static loading nor under a suddenly applied load. However, it is attainable by means of a specially tailored dynamic actuation, for example, by a two step voltage signal that is considered here. For this case, we map the conditions allowing the trapping of the beam in the second stable state, depending on the properties of the signal and the level of damping. We also demonstrate that trapping the dynamically bistable beam at a stable state located in the close proximity to the electrode may result in much more efficient gap usage than in the case of statically bistable beam or of an initially straight beam.

1. Introduction

Bistable structures are structures that may exhibit the existence of two different stable equilibria under the same loading. The transition between the two stable states in these structures is commonly referred to as snap-through buckling. Micro and nano electromechanical systems (MEMS and NEMS) incorporating bistable structural elements have functional advantages in applications such as switches [1,2], sensors [3,4] and non-volatile memories [5]. These applications stimulated a renewed interest in the rich mechanics of bistable beams [6–15] with special focus on the influence of the electrostatic force whose nonlinearly depends on the beam deflection [16–19].

While large body of works is focused on the beam quasi-static behavior, the snap-through and pull-in transitions are both essentially dynamic phenomena. The dynamic effects become especially paramount under transient loading conditions. The dynamics of electrostatically operated micro beams were studied in [8,20–33] for various types of actuation profiles. In [21], a dynamically actuated bistable micro truss was explored, revealing the feasibility of switching under a low voltage.

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A numerical study incorporating the response behavior of an initially curved undamped beam under a suddenly applied step load was carried out in [8]. A study of a damped initially curved beam under a suddenly applied step signal was carried out in [28], using one degree of freedom reduced order (RO) model of the device. The conditions of dynamic instabilities for such a case were formulated, and it was shown that the initial elevation of the beam required for the dynamic snap-through is higher than in the case of quasi-static loading. The critical dynamic snap-through and dynamic pull-in values of the voltage parameters are consistently lower, and critical deflections are higher than their static counterparts. In [31], various possible types of responses to a mechanical harmonic loading were designated and mapped in terms of the loading parameters, in addition to a basin of attraction study. In [34], the analysis of a prestressed bistable beam actuated by harmonic electrostatic load was investigated. The beam behavior was investigated using phase plane analysis as well as by studying its frequency response, leading to a conclusion that one DOF is sufficient for capturing the main features of the beam response. Using a phase plane analysis, the basin of attraction of a cantilever beam was derived theoretically and experimentally in [35,36]. It was concluded that the beam is sensitive to initial conditions and that the boundaries of the basin of attraction are characterized by an erosion pattern.

One of the advantages of bistable beams is that they may demonstrate much larger gap usage. Namely the distance the curved beam travels prior to the pull-in collapse, is usually larger than in the case of an initially straight beam. For example, bistable devices were shown to use up to about 84% of the gap [6], as opposed to a straight beam which use up to 75% at comparable or higher voltage (a full review can be found in [37]). Note, that larger gap usage can be achieved for a straight beam by means of feedback control. For example, a study incorporating a cantilever device [20] has shown that bistable properties can be achieved by a closed-loop controlled actuation. The device is shown to use 83% of the gap as opposed to 45% usage in an open-loop actuation where the cantilever's equilibrium curve has one limit point (i.e. pull-in). The analysis showed various phase plane responses in which trajectories were observed orbiting both stable fixed points before convergence to one of them. However, realization of a feedback control in micro, and especially nano scale devices, is usually challenging.

In the present work, we investigate the behavior of two curved beam configurations under different loading scenarios. In one scenario, the response of a beam of specific configuration to a suddenly applied step actuation is studied. It is shown, that depending on the damping and actuation voltage, the equilibrium state that the beam converges to after the snap-through event, can be at its first equilibrium configuration (bouncing), at the second stable state (snap-through), or at the electrode (pull-in). However, there are beam configurations for which a suddenly applied step load, as well as a quasi-static load, cannot bring the beam to its second isolated stable equilibrium configuration. Hence, for the second scenario in this work, we consider a beam which is subjected to a two-step actuation. The analysis shows that a specially tailored, time dependent two step actuation, can bring the beam to this statically inaccessible second stable equilibrium. This gives way to the possibility of trapping the dynamically bistable beam at a stable configuration, which is very close to the electrode.

In both instances, the time history response obtained on a basis of a lumped RO model is used to build a map defining the characteristic parameters required for the beam to exhibit dynamic snap-through. A rich behavior is observed in the boundary between the areas which define the parameters required for the beam to rest at one of its stable configurations.

2. Model

2.1. Formulation

We consider a flexible initially curved, double clamped prismatic micro beam of length L having a rectangular cross-section A of width $\hat{b}$ and thickness $\hat{d}$, as shown in Fig. 1. The beam is assumed to be made of homogeneous isotropic linearly elastic material with Young's modulus E and density ρ. Since the width $\hat{b}$ of a micro-beam is typically larger than its thickness $\hat{d}$, an effective (plain strain) modulus of elasticity $\tilde{E} = E/(1-\nu^2)$ is used, where ν is Poisson's ratio. The initial shape of the stress-free beam, for $\hat{t} = 0$, is described by the function $\hat{w}(\hat{x}, 0) = \hat{w}_0(\hat{x}) = \hat{h}_0 z_0(\hat{x})$, where $\hat{h}_0$ is the initial elevation of the beam central point above its ends, and $z_0(\hat{x})$ is a non-dimensional function such that $\max_{\hat{x}\in[0,L]}\left[z_0(\hat{x})\right] = 1$.

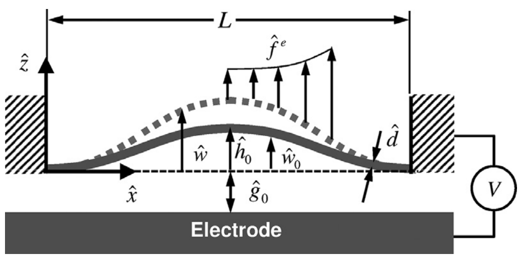

Fig. 1. Model of an initially curved beam actuated by a distributed electrostatic force $\hat{f}^e$. The black solid line represents the designed initial shape with initial nominal elevation $\hat{h}_0$. The upper gray dashed line corresponds to the deformed configuration. Positive directions of the beam deflection and of the loading are shown.