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Global bifurcations of a rotating pendulum with irrational nonlinearity

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ABSTRACT

In this paper, the authors consider a rotating pendulum under nonlinear perturbation which allows us to study various kinds of bifurcations and limit cycles. This system exhibits smooth or discontinuous dynamics depending on the value of a mechanical parameter. It is shown that the perturbed smooth system undergoes a pitchfork bifurcation, a homo–heteroclinic orbits transition, a single Hopf bifurcation, a double Hopf bifurcation, a pair of homoclinic bifurcations, a Hopf–homoclinic bifurcation and two saddle–node bifurcation of periodic orbits. The number, position and stability of all the oscillating and rotational limit cycles are given as the parameters vary. We find five limit cycles in the smooth case due to two saddle–node bifurcation of periodic orbits existing. Unlike the smooth case, the perturbed discontinuous system has a homoclinic-like bifurcation and without any saddle–node bifurcation of periodic orbits. Additionally, some results obtained herein bear significant similarities to the codimension-two bifurcation of duffing oscillator and SD oscillator, which may be helpful to explore the codimension bifurcation of the cylindrical pendulum system with irrational nonlinearity.

1. Introduction

The study of pendulum is one of the earliest scientific topics whose scientific importance has been an indisputable fact [1]. In the 16th century, Galileo found the isochronism of small angle oscillation through classical experiments, which marked the beginning of the scientific research of the pendulum. In the 17th century, Huygens discovered the periodicity of large swings deviating from the law of isochronism, which is one of the earliest records of nonlinear phenomena. By the middle of the 20th century, the interest in the study of pendulum has grown nearly in an exponential manner due to the invention of digital computers. Nowadays, its interest extends from science to civil, education [2], military [3] and industry [4] owing to the wide range of application.

Many kinds of pendulum-like systems have been presented and investigated by using experimental method, numerical method and analytical method over the past half century, which greatly enrich the research content of pendulum. For example, a new concept of using mechanical pendulum systems for wave energy extraction has been given in [5–7]. Synchronization of pendulum hanging on a common movable beam has been the subject of the research by a number of authors [8–10]. Chaotic dynamics in pendulum-like system has received a great deal of treatment [11–13]. Approximate analytical solutions for oscillatory and rotational motion of a parametric pendulum were discussed in [14]. The response of a

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spring–mass–damper system with an excited pendulum hinged to the mass was studied by using the harmonic balance method [15]. A two degree of freedom auto-parametric vibration absorber system in resonant excitation was considered by using the method of averaging [16]. The bifurcation in an inverted pendulum with the high frequency excitation was proved by using analytical and experimental investigations in [17]. Subharmonic and homoclinic bifurcations in a parametrically forced pendulum was studied in [18]. Codimension-two bifurcations in a pendulum with feedback control was investigated in [19]. Throughout the hot research topics of pendulum system, very few attempts have been made in studying the oscillating and rotational limit cycles of the autonomous system, which is of fundamental importance to reveal the essential phenomenology of pendulum from the view of nonlinear science and may be helpful to explore directly the codimension bifurcation of the cylindrical pendulum system avoiding Taylor's expansion.

In the past decade, a smooth and discontinuous (SD) oscillator whose nonlinearity can be smooth or discontinuous depending on the value of a smoothness parameter had been received much attention [20–22]. This SD oscillator consists of a lumped mass moving along a line and connected to a rigid frame by a pair of linearly elastic springs, whose codimension-two bifurcation including one saddle–node bifurcation of periodic orbits [23,24] was studied in [25]. In order to understand the effects of the coupling between smooth and non-smooth nonlinearities in dynamical systems, a simple mechanical model with non-regularized unilateral contact and Coulomb friction conditions was investigated in [26].

Recently, a rotating pendulum with irrational nonlinearity is proposed [27], which bears significant similarities to the conventional pendulum coupled with SD oscillator with the homoclinic orbits of the first type and second type [28]. The chaotic boundary of this rotating pendulum system have been obtained by constructing an approximate system successfully avoiding the barrier of the associated irrational nonlinearity [29]. The aim of this paper is to start exploring the global bifurcations of the rotating pendulum with irrational nonlinearity under the nonlinear perturbation $(\xi + \delta \cos x)\dot{x}$ inspired by an example from Guckenheimer and Holmes, the detail seen in [30].

The motivations and contributions of this paper are (i) to study the global bifurcations of the rotating pendulum with irrational nonlinearity, (ii) to display multiple dynamic bifurcations and their transitions as the parameters vary, (iii) to give the number, position and stability of the oscillating and rotational limit cycles, (iv) to find five limit cycles due to two saddle–node bifurcation of periodic orbits existing, (v) to understand the generation of the rotational limit cycle owing to the fracture of the homoclinic orbits of second type, all which reveal the strongly irrational nonlinearity and various kinds of cylindrical dynamics phenomena.

2. The rotating pendulum with irrational nonlinearity

Consider the rotating pendulum system shown in Fig. 1(a) and (b) which comprises a simple pendulum linked by an oblique spring with a rigid support. The equation of motion reads

$$mLx'' + mg\sin x + kh\sin x\left(1 - \frac{l}{\sqrt{L^2 + h^2 - 2Lh\cos x}}\right) = 0, \tag{1}$$

where the prime denotes the derivative with respect to time t, x is the angular displacement, m is the mass of the pendulum, g is the acceleration due to gravity, L is the rod length, k and l are the stiffness and relax length of the spring, h is the height from A to B.

Without loss of generality, system (1) can be made dimensionless by letting $\tau = \sqrt{(mg + kh)/(mL)}\,t$,

$$\ddot{x} + \sin x\left(1 - \frac{q}{\sqrt{1 + \lambda^2 - 2\lambda\cos x}}\right) = 0, \quad q = \frac{klh}{(mg + kh)L}, \quad \lambda = \frac{h}{L}, \tag{2}$$

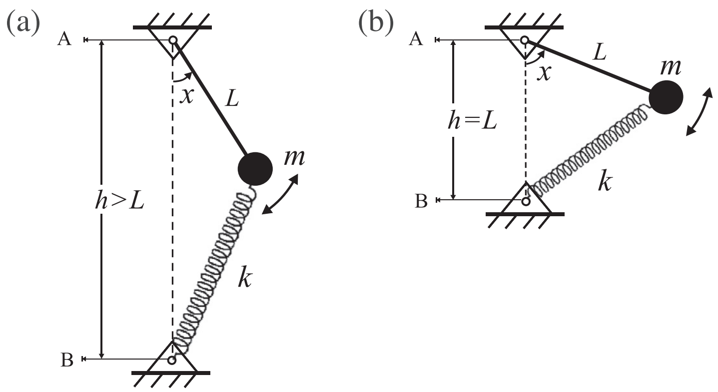

Fig. 1. The physical model of the rotating pendulum: (a) for $h > L$ and (b) for $h = L$.