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Analysis of stochastic effects in Kaldor-type business cycle discrete model

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ABSTRACT

We study nonlinear stochastic phenomena in the discrete Kaldor model of business cycles. A numerical parametric analysis of stochastically forced attractors (equilibria, closed invariant curves, discrete cycles) of this model is performed using the stochastic sensitivity functions technique. A spatial arrangement of random states in stochastic attractors is modeled by confidence domains. The phenomenon of noise-induced transitions "chaos-order" is discussed.

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1. Introduction

Modeling and analysis of the complex dynamic processes in economics attract attention of a wide range of researchers [1–5]. One of the earliest model of business cycles has been proposed by Kaldor [6]. Using nonlinear functions for the interaction of economic variables, he explained transitions from equilibrium to oscillatory dynamic regimes. A rigorous mathematical substantiation of the Kaldor approach was elaborated in [7] on the base of continuous-time dynamic model possessing stable limit cycles. Since these early papers, the Kaldor-type models were widely studied [8–13].

In [14,15], Kaldor's ideas were developed for discrete-time dynamic systems of two nonlinear difference equations. The Kaldor-type discrete models exhibit a much greater diversity of dynamic regimes including quasi-periodic and chaotic ones. Complex dynamics of the Kaldor's discrete-time deterministic model of business cycles was studied in [16] where local and global bifurcations, regular and chaotic attractors with non-trivial geometry of basins of attraction were investigated.

It is well-known that uncontrolled random disturbances are an inevitable attribute of any real system, especially in economics. Even weak noise in nonlinear system can considerably transform its dynamics. So, an analysis of the influence of random disturbances is a challenging problem of the modern theory of economic dynamics. Stochastic phenomena in continuous-time Kaldor model are actively studied [17–19]. In present paper, we first study discrete Kaldor model in the presence of random disturbances.

An interplay of the stochasticity and nonlinearity cause various phenomena, such as noise-induced transitions [20], stochastic bifurcations [21], stochastic resonance [22–25], noise-induced order and chaos [26–28].

For discrete systems with random disturbances, a full mathematical description of stochastic dynamics is given by Perron–Frobenius equation [29,30]. Since an analytical solution of this functional equation is possible only in very

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particular cases, stochastic effects are commonly studied by a direct numerical simulation. But this method is extremely time-consuming especially in the parametric analysis. So, it is very important to find constructive asymptotics and approximations. In [31], an approach based on the stochastic sensitivity functions (SSF) technique has been proposed for the approximation of the probability distributions near stochastically forced equilibria and cycles of discrete-time systems. Constructive abilities of this technique have been demonstrated in the analysis of noise-induced phenomena in one-dimensional [32,33] and two-dimensional [31] models. In the paper [34], SSF technique was developed for systems with closed invariant curves.

In present paper, we suggest a general approach to the analysis of a variety of complex regimes of stochastic nonlinear dynamics in Kaldor-type discrete model.

The paper is organized as follows. In Section 2, we shortly discuss dynamical regimes and bifurcations in the initial deterministic model. Here, along with equilibria and discrete cycles, we study closed invariant curves which appear due to Neimark–Sacker bifurcation. New results concerning the angular distribution of states in closed invariant curves are presented.

In Section 3, the nonlinear stochastic dynamics of the randomly forced Kaldor model is studied. Our approach to the numerical analysis of the stochastic attractors is based on SSF technique and confidence domains method. Here, we carry out a parametric analysis of dispersions of random states around stable equilibria, closed invariant curves and discrete cycles. We also consider the stochastic phenomena caused by noise-induced transitions between discrete 4-cycle and chaotic attractor. Using Lyapunov exponents, we study the transformation “chaos-order”.

Details of the SSF technique and confidence domains method are shortly presented in Appendix.

2. Deterministic model

Consider a discrete Kaldor model [16]:

$$\begin{aligned} y_{t+1} &= y_t + \alpha\,(I(y_t,k_t) - S(y_t)), \\ k_{t+1} &= (1-\delta)k_t + I(y_t,k_t). \end{aligned} \tag{1}$$

Here, y is a national income, k is a capital, $I(y, k)$ and $S(y)$ are investment and saving functions [35]:

$$I(y,k) = \sigma\mu + \gamma\left(\frac{\sigma\mu}{\delta} - k\right) + \arctan(y-\mu), \quad S(y) = \sigma y.$$

Dynamics of this system is defined by the following positive parameters. The control parameter α represents a speed of adjustment, the coefficient δ represents capital stock's depreciation rate, σ is a propensity to save, μ is a normal level to income, γ is a feedback coefficient.

Coordinates $\bar{y}$, $\bar{k}$ of the equilibrium of the model (1) satisfy the following system:

$$y = \mu + \frac{\delta}{\sigma(\delta+\gamma)}\arctan(y-\mu), \quad k = \frac{\sigma}{\delta}y. \tag{2}$$

For any parameters, the system (1) has the equilibrium $M(\mu, \mu\frac{\sigma}{\delta})$. In present paper, we consider the case $\sigma > \frac{\delta}{\delta+\gamma}$ when this equilibrium is unique.

The Jacobi matrix of the map (1) can be written as

$$F = \begin{bmatrix} 1 - \alpha\sigma + \frac{\alpha}{1+(y-\mu)^2} & -\alpha\gamma \\ \frac{1}{1+(y-\mu)^2} & 1-\delta-\gamma \end{bmatrix}.$$

In what follows, we fix parameters $\sigma = 0.8,\ \gamma = 0.6,\ \delta = 0.2,\ \mu = 10,$ and study a dynamics of the Kaldor model depending on the control parameter α. For this set of parameters, the equilibrium P has coordinates $\bar{y} = 10, \bar{k} = 40,$ and the corresponding Jacobi matrix is

$$F = \begin{bmatrix} 1+0.2\alpha & -0.6\alpha \\ 1 & 0.2 \end{bmatrix}.$$

The system (1) possesses both regular and chaotic attractors, and demonstrates various types of bifurcations. In Fig. 1, k-coordinates of attractors are plotted by grey color for the interval $0 < \alpha < 4.5$. Here, the largest Lyapunov exponent $\Lambda(\alpha)$ is also shown by black color. Zones $\Lambda > 0$ correspond to the chaotic dynamics.

Here, one can specify some typical zones. For $0 < \alpha < 0.3095$, the system exhibits the node-type stable equilibrium (see Fig. 2a). This equilibrium is a focus for $0.3095 < \alpha < 1.25$ (see Fig. 2b). At the point $\alpha = 1.25$, the Neimark–Sacker bifurcation [36,37] occurs.

In zones $1.25 < \alpha < 1.976$, $2.026 < \alpha < 2.51$, attractors of the system (1) are closed invariant curves (see solid lines in Fig. 2c for $\alpha = 1.4,\ 1.7,\ 2.3$). As the parameter α increases, closed invariant curves are destroyed and transformed into discrete cycles, which further are transformed into new closed invariant curves again. In the zone $1.976 < \alpha < 2.026$, stable 6-cycles are observed (see dots in Fig. 2c for $\alpha = 2$). In the zone $2.65 < \alpha < 3.33$, the system exhibits stable 4-cycles. For $2.65 < \alpha < 2.75$, the system is bistable. Here, a stable discrete 4-cycle coexists with another attractor. For such short

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