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# Adaptation of quadtree meshes in the scaled boundary finite element method for crack propagation modelling

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ABSTRACT

A crack propagation modelling technique combining the scaled boundary finite element method and quadtree meshes is developed. This technique automatically satisfies the compatibility requirement between adjacent quadtree cells irrespective of the presence of hanging nodes. The quadtree structure facilitates efficient data storage and rapid computations. Only a single cell is required to accurately model the stress field near crack tips. Crack growth is modelled by splitting the cells in the mesh into two. The resulting polygons are directly modelled by the scaled boundary formulation with minimal changes to the mesh. Four numerical examples demonstrate the salient features of the technique.


## 1. Introduction

The quadtree decomposition [1] is a hierarchical-type data structure, in which each parent is recursively divided into four children. Quadtree algorithms derived from this quaternary tree structure are particularly efficient for data storage and retrieval. The efficiency of these algorithms make them particularly popular in applications that demand high speed and efficiency such as computer graphics [2,3] and image processing [4]. In computational mechanics, quadtree algorithms are usually used in large scale simulations typical in the modelling of earthquake and ground motions [5], flood [6] and tsunami [7].

In solid mechanics, where the finite element method is most widely used, the quadtree decomposition is an intermediate step that is usually employed in the automatic mesh generation of triangular meshes [8,9] and quadrilateral meshes [10,11]. Mesh generators employing quadtree algorithms are fast, efficient and are capable of achieving rapid and smooth transitions of element sizes between regions of mesh refinement. Adaptation of quadtree meshes for direct computational analyses within the framework of the finite element method, however, is not widespread. The primary reason for this is the presence of hanging nodes (shown as the filled circles in Fig. 1) between two adjacent elements of different sizes. The presence of hanging nodes destroys the displacement compatibility between the adjacent elements, prohibiting them to be directly used with quadtree meshes.

Several methods have been proposed in the literature to resolve the displacement incompatibility introduced by the hanging nodes so that quadtree meshes can be used in finite element computations.These include constraining the displacements of the hanging nodes through Lagrange multipliers or penalty methods e.g. [5], constraining the hanging nodes to the corner nodes and adding temporary triangular or rectangular elements [12]. These approaches can only be used when the

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**Nomenclature**

| | |
|---|---|
| $\mathbf{B}$ | scaled boundary finite element strain–displacement matrix |
| $\mathbf{D}$ | material constitutive matrix |
| $\mathbf{K}$ | stiffness matrix |
| $\mathbf{N}$ | shape function matrix |
| $\mathbf{S}$ | Schur/eigenvalue matrix |
| $\mathbf{Z}$ | Hamiltonian matrix |
| $\mathbf{c}$ | integration constants |
| $\mathbf{q}$ | internal force vector |
| $\mathbf{u}$ | displacement vector |
| $K$ | stress intensity factors |
| $\mathbf{\Psi}$ | transformation matrix |
| $\mathbf{\Psi}_\sigma$ | stress mode |
| $\theta$ | angle |
| $\lambda$ | eigenvalue |
| $\xi$ | radial coordinate |
| $\eta$ | local coordinate |
| $d_\Omega$ | signed distance function |
| $s_\Omega$ | sign function |
| $\Omega$ | domain |
| $\partial\Omega$ | boundary of domain |
| $\lvert\mathbf{f}\rvert$ | crack propagation length |

quadtree mesh satisfy the 2:1 rule i.e. the maximum difference between two adjacent elements is less than one [8]. Constraint equations have to be properly constructed to treat the displacement incompatibility when the 2:1 rule is not satisfied e.g. Ainsworth et al. [13].

An alternate approach that enables quadtree meshes to be directly used in finite element computations is by developing special transitional finite elements for the cells in a quadtree mesh that contain hanging node(s) e.g. [14–16]. Gupta [14] derived the shape functions for quadrilateral finite elements containing hanging nodes that are located at the midpoints of the element edges by modifying the shape functions of the corner nodes of the standard quadrilateral elements. Transition finite elements can also be derived using the assumed stress hybrid method of Pian [17]. Within this approach, several element designs are possible; as outlined by Lo et al. [15] and Sze and Wu [16]. Alternative approaches that can be used to construct transition elements include using incompatible modes [18].

The use of quadtree meshes within the framework of the extended finite element method [19] to model problems involving fracture and discontinuous surfaces has also been reported in the literature. Fries et al. [20] reported two approaches to adapt quadtree meshes into the extended finite element method. The first approach uses the transition elements developed by Gupta [14]. The second approach constrains the displacements of the hanging nodes to be the average of the neighbouring corner nodes and is similar to the approach adopted by Bielak et al. [5].

Sukumar and Tabarraei [21] outlined a technique to develop conforming polygon elements having arbitrary number of sides. The shape functions of the polygon elements can be derived, for example, from Wachspress interpolants, mean value

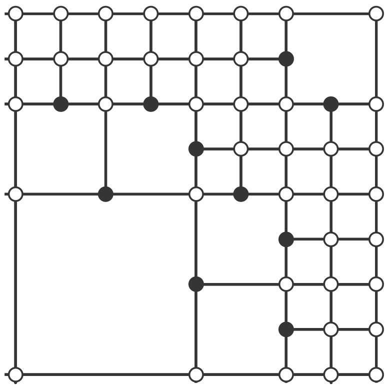


**Fig. 1.** Hanging nodes in a generic quadtree mesh.