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Robustness analysis for parameter matrices of global exponential stable stochastic time varying delay systems

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ABSTRACT

In this paper, we analyze the robustness of global exponential stable stochastic delayed systems subject to the uncertainty in parameter matrices. Given a globally exponentially stable systems, the problem to be addressed here is how much uncertainty in parameter matrices the systems can withstand to be globally exponentially stable. The upper bounds of the parameter uncertainty intensity are characterized by using transcendental equation for the systems to sustain global exponential stability. Moreover, we prove theoretically that, the globally exponentially stable systems, if additive uncertainties in parameter matrices are smaller than the upper bounds arrived at here, then the perturbed systems are guaranteed to also be globally exponentially stable. Two numerical examples are provided here to illustrate the theoretical results.

1. Introduction

Time delays are often encountered in various practical systems such as chemical processes, neural networks and long transmission lines in pneumatic systems [1,2]. It has been shown that the existence of time-delays may lead to oscillation, divergence, instability, greatly increasing the difficulty of stability analysis and control design. Many researchers in the field of control theory and engineering study the robust control of time-delay systems. The main methods of stability analysis can be classified into two types: frequency-domain and time-domain. The former use the sum of squares technique. As to the time-domain approach, Lyapunov functional is a powerful tool, which can deal with time varying delays.

For most successful applications of the systems, the stability is usually a prerequisite. The stability of the systems depends mainly on their parametrical configuration. Moreover, in the applications of the systems, external random disturbances and time delays are common and hardly avoided. It is known that random disturbances and time delays in the systems may result in oscillation or instability of the nonlinear systems. The stability analysis of the delayed systems and the systems with external random disturbances has been widely investigated in recent years (see, e.g., [3–8], and the references cited therein).

In practice, when we estimate systems parameter matrices, there are always some uncertainty and errors. For the parameter matrices uncertainty, two types are studied widely: time varying structured uncertainty [9–12] and polytopic-type uncertainty [13–15]. If the uncertainty is too large, the stable systems may becomes instable, the intensity of parameter matrices uncertainty is often the sources of instability and they can destabilize stable delay systems if it exceeds its limits. The instability depends on the intensity of parameter matrices uncertainty. Many people analyze the robust stability of parameter uncertainty systems for given structured uncertainty or polytopic-type uncertainty. For a stable delay system, if the intensity of parameter matrices uncertainty is low, the perturbed delay system may still be stable. Therefore, it is interesting to determine how much parameter matrices uncertainty of stable delay systems can withstand without losing global

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exponential stability. Shen et al. [10] investigated the robustness of global exponential stability of recurrent neural networks in the presence of time delays and random disturbances. Although the various stability properties of stable delay systems have been extensively investigated using the Lyapunov and the linear matrix inequality methods, the robustness of the global stability for parameter matrices of systems is rarely analyzed directly by estimating the upper bounds of parameter matrices uncertainty level.

Motivated by the above discussion, our aim in this paper is directly to quantify the parameter uncertainty level for stable systems only use the definition of stability. That is, we characterize the robustness for parameter matrices in general form of systems by deriving the upper bounds of parameter matrices uncertainty for global exponential stability. We prove theoretically that, for globally exponentially stable systems, if additive parameter matrices uncertainty is smaller than the derived upper bounds herein, then the systems are guaranteed to be globally exponentially stable.

The remainder of this paper is organized as follows. Section 2 provides problem formulation. Section 3 discusses the impact of uncertainty in parameter matrices for the global exponential stability of systems. In Section 4, two numerical examples are given to illustrate the theoretical results. Finally, concluding remarks are given in Section 5.

2. Problem formulation

Throughout this paper, unless otherwise specified, R^n and $R^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of $n\times m$ real matrices. Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geqslant 0},P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). $\omega(t)$ be a scalar Brownian motion defined on the probability space. If A is a matrix, its operator norm is denoted by $\|A\| = \sup\{|Ax| : |x| = 1\}$, where $|\cdot|$ is the Euclidean norm. Denote $L^2_{\mathcal{F}_0}([-\bar{\tau},0];R^n)$ as the family of all $\mathcal{F}_0-$ measurable $C([-\bar{\tau},0];R^n)$ valued random variables $\psi=\{\psi(\theta): -\bar{\tau}\leqslant\theta\leqslant 0\}$ such that $\sup_{-\bar{\tau}\leqslant\theta\leqslant 0}E|\psi(\theta)|^2<\infty$ where $E\{\}$ stands for the mathematical expectation operator with respect to the given probability measure P.

Consider a stochastic delayed systems

$$\begin{aligned} &dx(t) = [Ax(t) + Bx(t-\tau(t))]dt + [W_1x(t) + W_2x(t-\tau(t))]d\omega(t), \quad t > t_0 \\ &x(t) = \psi(t-t_0) \in L^2_{\mathcal{F}_0}([t_0-\bar{\tau},t_0];R^n), \quad t_0-\bar{\tau}\leqslant t\leqslant t_0 \end{aligned} \tag{1}$$

where $x(t) = (x_1(t),\ldots,x_n(t))^T \in R^n$ is the state vector of the system, $t_0\in R_+$ and $\psi\in R^n$ are the initial values, $A\in R^{n\times n}, B\in R^{n\times n}$ are parameter matrices, W_1, W_2 are $R^{n\times n}$ matrices, which stand for noise function. We assume that origin is an equilibrium point of (1), $\tau(t)$ is a time varying delay that satisfies $\tau(t):[t_0,+\infty)\to[0,\bar{\tau}], \tau'(t)\leqslant\mu<1, \psi=\{\psi(s): -\bar{\tau}\leqslant s\leqslant 0\}\in C([-\bar{\tau},0],R^n)$, $\bar{\tau}$ is the maximum of delay.

Now we define the global exponential stability of the state of systems (1).

Definition 1 [16]. Systems (1) is said to be almost sure globally exponentially stable if for any $t_0\in R_+, \psi\in L^2_{\mathcal{F}_0}([-\bar{\tau},0];R^n)$, there exist $\alpha>0$ and $\beta>0$ such that $\forall t\geqslant t_0, |x(t;t_0,\psi)|\leqslant\alpha\|\psi\|\exp(-\beta(t-t_0))$ hold almost surely; i.e., the Lyapunov exponent $\limsup_{t\to\infty}(\ln|x(t;t_0,\psi)|/t)<0$ almost surely, where $x(t;t_0,\psi)$ is the state of system (1). Systems (1) is said to be mean square globally exponentially stable if for any $t_0\in R_+, \psi\in L^2_{\mathcal{F}_0}([-\bar{\tau},0];R^n)$, there exist $\alpha>0$ and $\beta>0$ such that $\forall t\geqslant t_0, E|x(t;t_0,\psi)|^2\leqslant\alpha\|\psi\|\exp(-\beta(t-t_0))$ hold; i.e., the Lyapunov exponent $\limsup_{t\to\infty}(\ln(E|x(t;t_0,\psi)|^2)/t)<0$, where $x(t;t_0,\psi)$ is the state of systems (1).

From the definition, it is clear that the almost sure global exponential stability of systems (1) implies the mean square global exponential stability of systems (1) [16], but not vice versa. However, as (1) is a linear system, we have the following lemma [16, Theorem 6.2, pp. 175].

Lemma 1. *The global exponential stability in sense of mean square of systems (1) implies the almost sure global exponential stability of systems (1).*

Numerous criteria for ascertaining the global exponential stability of systems (1) have been developed; e.g., [10,11,15] and the references therein.

3. Main results

Now, the question is given a globally exponentially stable stochastic system, how much the parameter uncertainty intensity of parameter matrices the systems can bear? We first consider the parameter uncertainty intensity adding to parameter matrix A, the perturbed systems changes as

$$\begin{aligned} &dy(t) = [(A+\Delta A)y(t) + By(t-\tau(t))]dt + [W_1y(t) + W_2y(t-\tau(t))]d\omega(t), \quad t > t_0 \\ &y(t) = \psi(t-t_0) \in L^2_{\mathcal{F}_0}([t_0-\bar{\tau},t_0];R^n), \quad t_0-\bar{\tau}\leqslant t\leqslant t_0 \end{aligned} \tag{2}$$

where the notations of (2) are the same as in Section 2, ΔA stands for parameter matrix uncertainty intensity.