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Permanence and periodic solutions for an impulsive reaction–diffusion food-chain system with ratio-dependent functional response

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ABSTRACT

An impulsive reaction–diffusion periodic food-chain system with ratio-dependent functional response is investigated in the present paper. Sufficient conditions for the ultimate boundedness and permanence of the food-chain system are established based on the comparison theory of differential equation and upper and lower solution method. By constructing appropriate auxiliary function, the conditions for the existence of a unique globally stable positive periodic solution are also obtained. Some numerical examples are presented to verify our results. A discussion is given in the end of the paper.

1. Introduction

Reaction–diffusion equations can be used to model the spatiotemporal distribution and abundance of organisms. A typical form of reaction–diffusion population model is

$$\frac{\partial u}{\partial t} = \mathcal{D}\Delta u + uf(x,u),$$

where $u(x,t)$ is the population density at a space point x and time t, $\mathcal{D} > 0$ is the diffusion constant, Δu is the Laplacian of u with respect to the variable x, and $f(x,u)$ is the growth rate per capita, which is affected by the heterogeneous environment. Such an ecological model was first considered by Skellam [13], and similar reaction–diffusion biological models were also studied by Fisher [5] and Kolmogoroff et al. [7] earlier. In the past two decades, the reaction–diffusion models, especially in population dynamics, have been studied extensively. For example, Ainseba and Aniţa [1] considered a 2×2 system of semilinear partial differential equations of parabolic-type to describe the interactions between a prey population and a predator population. Meanwhile, they obtained some necessary and sufficient conditions for the stabilizability. Recently, xu and Ma [20] studied a reaction–diffusion predator–prey system with nonlocal delay and Neumann boundary conditions. They have established some sufficient conditions on the global stability of the positive steady state and the semi-trivial steady state. Liu and Huang [10] investigated a diffusive predator–prey model with Holling type III functional response and obtained some sufficient conditions for the ultimate boundedness of solutions and permanence of the system, they also

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presented the existence of a unique globally stable periodic solution. More researches on the reaction–diffusion population dynamical systems, see [16,4,19,6].

There are many examples of evolutionary systems which at certain instants are subjected to rapid to changes. In the simulations of such processes it is frequently convenient and valid to neglect the durations of rapid changes. The perturbations are often treated continuously. In fact, the ecological systems are often affected by environmental changes and other human activities. These perturbations bring sudden changes to the system. Systems with such sudden perturbations referring to impulsive differential equations, which have attracted the interest of many researchers in the past twenty years since they provide a natural description of several real processes. Process of this type is often investigated in various fields of science and technology, physics, population dynamics [3,11], epidemics [21], ecology, biology, optimal control [8], and so on.

Recently, some impulsive reaction–diffusion predator–prey models have been investigated. Especially, Akhmet et al. [2] presented an impulsive ratio-dependent predator–prey system with diffusion, meanwhile, they obtained some conditions for the permanence of the predator–prey system and for the existence of a unique globally stable periodic solution. Wang et al. [18] generalized the above impulsive ratio-dependent system to an $n+1$ species and get some analogous results. However, food-chain systems exist extensively in the population dynamics, and the reaction–diffusion food-chain systems have rarely been studied by scholars. Hence, the study of the dynamics on the impulsive reaction–diffusion food-chain system is the aim of this paper.

Motivated by the above works, we consider the following impulsive reaction–diffusion food-chain system with ratio-dependent functional response in this paper:

$$\frac{\partial u_1}{\partial t} = \mathcal{D}_1 \Delta u_1 + u_1\left(a_1(t,x) - b_1(t,x)u_1 - \frac{c_1(t,x)u_2}{u_1 + r_1(t,x)u_2}\right), \tag{1}$$

$$\frac{\partial u_2}{\partial t} = \mathcal{D}_2 \Delta u_2 + u_2\left(a_2(t,x) - b_2(t,x)u_2 + \frac{\tilde{c}_1(t,x)u_1}{u_1 + r_1(t,x)u_2} - \frac{c_2(t,x)u_3}{u_2 + r_2(t,x)u_3}\right), \tag{2}$$

$$\frac{\partial u_3}{\partial t} = \mathcal{D}_3 \Delta u_3 + u_3\left(-a_3(t,x) + \frac{\tilde{c}_2(t,x)u_2}{u_2 + r_2(t,x)u_3}\right), \tag{3}$$

$$u_i(t_k^+, x) = u_i(t_k, x) f_k^i(x, u_1(t_k,x), u_2(t_k,x), u_3(t,x)), \quad k = 1,2,\ldots, \tag{4}$$

$$\left.\frac{\partial u_i}{\partial n}\right|_{\partial\Omega} = 0, \quad i = 1,2,3. \tag{5}$$

In the system, it is assumed that the predator and prey species are confined to a fixed bounded space domain $\Omega \subset \mathbb{R}^n$ with smooth boundary $\partial\Omega$ and non uniformly distributed in the domain. Furthermore, they are subjected to short-term external influence at fixed moment of time t_k, where $\{t_k\}$ is a sequence of real numbers $0 = t_0 < t_1 < \cdots < t_k < \cdots$ with $\lim_{k\to\infty} t_k = +\infty$. Denote by $\partial/\partial n$ the outward derivative, $\bar{\Omega} = \Omega \cup \partial\Omega$, and $\Delta u = \partial^2 u/\partial x_1^2 + \cdots + \partial^2 u/\partial x_n^2$ the Laplace operator.

In Eqs. (1)–(3), $\mathcal{D}_1\Delta u_1,\ \mathcal{D}_2\Delta u_2$ and $\mathcal{D}_3\Delta u_3$ with positive diffusion coefficients $\mathcal{D}_1,\ \mathcal{D}_2$ and $\mathcal{D}_3$ reflect the non-homogeneous dispersion of populations. Neumann boundary conditions (5) characterize the absence of migration. In the absence of predator, the prey species has a logistic growth rate. We assume that the predator functional responses have the form of the ratio-dependent functional response functions $c_i(t,x)u_{i+1}/(u_i + r_i(t,x)u_{i+1}),\ i = 1,2,$ which reflect the capture ability of the predators.

In this paper, we will investigate the asymptotic behavior of non-negative solutions for impulsive reaction–diffusion system (1)–(5). Note that according to biological interpretation of the solutions $u_1(t,x),\ u_2(t,x)$ and $u_3(t,x)$ they must be non-negative. We will give conditions for the long-term survival of each species in terms of permanence. The permanence of the system indicates that the number of individuals of each species stabilizes in certain boundaries with time.

This paper is organized as follows. In Section 2, we give some basic assumptions and useful auxiliary results. Conditions for the ultimate boundedness of solutions and permanence of the system are obtained in Section 3. In Section 4, we establish conditions for the existence of the unique periodic solution of the system. Examples and numerical simulations are presented in Section 5 to verify the feasibility of our results. Finally, we discuss our results and present some interesting problems.

2. Preliminaries

Let $\mathbb{N}$ and $\mathbb{R}$ be the sets of all positive integers and real numbers, respectively, and $\mathbb{R}_+ = [0,\infty)$. The following assumptions will be needed throughout the paper.

- (A_1) Functions $a_i(t,x),\ b_i(t,x),\ c_i(t,x),\ \tilde{c}_i(t,x),\ r_i(t,x),\ i = 1,2$ and $a_3(t,x)$ are bounded positive-valued on $\mathbb{R} \times \bar{\Omega}$, continuously differentiable in t and x, periodic in t with a period $\tau > 0$;
- (A_2) Functions $f_k^i(x,u_1,u_2,u_3),\ i = 1,2,3,\ k \in \mathbb{N}$, are continuously differentiable in all arguments and positive-valued;
- (A_3) There exists a number $p \in \mathbb{N}$ such that $t_{k+p} = t_k + \tau$ for all $k \geqslant 1$;