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Persistence and global stability of Bazykin predator–prey model with Beddington–DeAngelis response function

Sahabuddin Sarwardi [a], Mainul Haque [b], Prashanta Kumar Mandal [c,*]

[a] *Department of Mathematics, Aliah University, DN-41, Sector-V, Salt Lake City, Kolkata 700 091, West Bengal, India*
[b] *University Division of Anaesthesia & Intensive Care, School of Clinical Sciences, Queen's Medical Centre (Nottingham University Hospital NHS Trust), University of Nottingham, Derby Road, Nottingham NG7 2UH, UK*
[c] *Department of Mathematics, Visva-Bharati, Santiniketan 731 235, West Bengal, India*

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ABSTRACT

In this article, a predator–prey model of Beddington–DeAngelis type with discrete delay is proposed and analyzed. The essential mathematical features of the proposed model are investigated in terms of local, global analysis and bifurcation theory. By analyzing the associated characteristic equation, it is found that the Hopf bifurcation occurs when the delay parameter τ crosses some critical values. In this article, the classical Bazykin's model is modified with Beddington–DeAngelis functional response. The parametric space under which the system enters into Hopf bifurcation for both delay and non-delay cases are investigated. Global stability results are obtained by constructing suitable Lyapunov functions for both the cases. We also derive the explicit formulae for determining the stability, direction and other properties of bifurcating periodic solutions by using normal form and central manifold theory. Our analytical findings are supported by numerical simulations. Biological implication of the analytical findings are discussed in the conclusion section.

1. Introduction

Ecological models have received much attention from scientists. It is well known that the dynamical behaviours, stability, interactivity, persistence, periodic oscillation, bifurcation and chaos of population with time delay have become a subject of extensive research activities, especially, the properties of periodic solutions are most impressive. Relevant references in this context are also vast and we shall therefore mention here a few of them (cf. Anderson [5], Bailey [6] and Diekmann et al. [15]). It is well understood that many of the processes both natural and man made in biology and medicine involve time delays. Time delays occur often in almost every situation, ignoring them, therefore, is not realistic. Kuang [27] mentioned that the animal must take time delays to digest their food before their further activities take place. Hence models of species dynamics without delays is an approximation at best rather it is more realistic to assume that the reproduction of predator after predating the prey will not be instantaneous, but mediated through some time lag which is required for gestation of the predator. Detailed arguments on the importance of time delays in realistic models can be found in the classical books of Gopalsamy [18], Kuang [27] and MacDonald [29]. So incorporation of time delay makes the predator–prey interactions one step closer to reality.

Mathematical model is an important tool in analyzing the ecological models. Ecological problems are challenging and important issues from both the ecological and the mathematical point of view (cf. Anderson and May [4], Beretta and Kuang

* Corresponding author. Tel./fax: +91 3463261029.
E-mail addresses: s.sarwardi@gmail.com (S. Sarwardi), mainul.haque@rediffmail.com (M. Haque), prashantakumar.mandal@visva-bharati.ac.in (P.K. Mandal).

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[9], Freedman [17], Hadeler and Freedman [20], Hethcote et al. [24], Ma and Takeuchi [28], Venturino [34], Xiao and Chen [36]). The dynamic relationship between predator and its prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. The traditional predator–prey models have been studied extensively (cf. Cantrell and Cosner [11], Cosner et al. [12], Cui and Takeuchi [13], Huo et al. [25] and Hwang [26]), but those are questioned by several biologists. Thus, the Lotka–Volterra type predator–prey model with the Beddington- DeAngelis functional response has been proposed and well studied. The model can be expressed as follows:

$$\frac{dx}{dt} = rx - \frac{bxy}{A + x + By} - ex^2, \tag{1}$$

$$\frac{dy}{dt} = y\left(-c + \frac{pbx}{A + x + By} - hy\right), \tag{2}$$

with the initial conditions $x(0) = x_0 > 0$ and $y(0) = y_0 > 0$. The functional response in system (1)–(2) was introduced by Beddington [8] and DeAngelis et al. [14]. It is similar to the well-known Holling type-II functional response but has an extra term By in the denominator which models mutual interference between predators. It can be derived mechanistically from considerations of time utilization (cf. Beddington [8]) or spatial limits on predation.

This paper is summarized as follows: The basic assumptions and our model are proposed in Section 2. Section 3 deals with some preliminary results like, boundedness of the solutions. We discuss the stability of the axial equilibrium and interior equilibrium with delay case in Section 6. The main results of our investigations are the stability and bifurcations connected with the nontrivial equilibria of the system are devoted in Sections 7 and 8. Simulation results are reported in Section 9, while a final discussion and interpretations of our results in terms of ecology are given in the concluding Section 10.

2. Basic assumptions and our mathematical model

In recent times, there are growing explicit biological and physiological evidences that in many situations when predator have to search for food, a more suitable general predator–prey theory should be based on so-called ratio-dependent theory in Akcakaya et al. [1], Arditi et al. [2], Arditi and Saiah [3] and Gutierrez [19], which may be roughly stated as that the per capita predator growth rate should be function of predator–prey abundance. The goal of this investigation is to study the dynamical properties of predator–prey model of Beddington–DeAngelis type with gestation delay. The functional response of Beddington–DeAngelis type was introduced by Beddington [8] and DeAngelis et al. [14] independently. It is similar to the well known Holling type-II functional response but has an extra term in the denominator, which models mutual interference between predators (cf. Cosner et al. [12]).

We assume that the reproduction of predator population after predating the prey will not be instantaneous, but mediated by some constant time lag $\tau > 0$ for gestation of predator, Wang and Chen [35] and Zhao et al. [37], here τ represents the time lag required for gestation of predator which is based on the assumption that the change rate of predators depends on the number of prey and of predators present at some previous time, p $(0 < p < 1)$ is the conversion factor and it represents the rate of conversion of the consumed prey into predator.

With the above assumptions the predator–prey model with Beddington–DeAngelis type functional response takes the following form:

$$\frac{dx}{dt} = rx - \frac{bxy}{A + x + By} - ex^2, \tag{3}$$

$$\frac{dy}{dt} = -cy + \frac{pbx(t-\tilde{\tau})y(t-\tilde{\tau})}{A + x(t-\tilde{\tau}) + By(t-\tilde{\tau})} - hy^2, \tag{4}$$

with the initial conditions $x(0) = x_0 > 0$ and $y(0) = y_0 > 0$.

By making use of the following transformation given by $x = \frac{ru}{e}, y = \frac{rpv}{e}$ and $T = rt$, then above dynamical system (3)–(4) is reduced to the following non-dimensional system as

$$\frac{du}{dt} = u - \frac{\epsilon uv}{1 + \alpha u + \beta v} - u^2, \tag{5}$$

$$\frac{dv}{dt} = -\gamma v + \frac{\epsilon u(t-\tau)v(t-\tau)}{1 + \alpha u(t-\tau) + \beta v(t-\tau)} - \delta v^2, \tag{6}$$

with $u(0) = u_0 > 0, v(0) = v_0 > 0$, where $\tau = r\tilde{\tau}, \alpha = \frac{r}{Ae}, \delta = \frac{hp}{e}, \beta = \frac{Brp}{Ae}, \gamma = \frac{c}{r}, \epsilon = \frac{pb}{Ae}$ (using 't' instead of 'T' for notational convenience).

3. Preliminary results

In this section we present some preliminaries, but important results including existence and uniqueness, conservativeness, dissipativeness, permanence, boundedness of solutions of the system (5)–(6). We may easily observe that if J be the