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Improved delay-dependent stability criteria for continuous system with two additive time-varying delay components ☆

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ABSTRACT

This paper investigated the problem of improved delay-dependent stability criteria for continuous system with two additive time-varying delay components. Free weighting matrices and convex combination method are not involved, which achieves much less numbers of linear matrix inequalities (LMIs) and LMIs scalar decision variables. By taking advantage of integral inequality and new Lyapunov–Krasovskii functional, new less conservative delay-dependent stability criterion is derived. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

In the past few decades, the stability of delayed systems have found successful applications in many areas such as signal processing, pattern recognition, associative memories, parallel computation, optimization solvers and so on. Many important results on the dynamical behaviors have been reported, for the recent progress, the reader is referred to [1–6] and references therein. It is known that, according to dependence on the size of the delays, developed stability and stabilization criteria are often classified into two categories: delay-independent stability criteria [7,8] and delay-dependent stability criteria [9–23]. The delay-independent stability criteria can guarantee the stability of the system irrespective of the size of time-delay. Due to undesirable dynamic network behaviors such as oscillation and instability, the delay-dependent stability criteria are concerned with the size of delay and provide an upper bound of time-delay size which assure the asymptotic stability of the system. Therefore, it is important to analysis the stability of delayed continuous system in the literature.

In recent years, the following basic mathematical model has been widely studied [3–6]:

$$\dot{x}(t) = Ax(t) + A_h x(t - \tau(t)),$$

where $\tau(t)$ is a time delay in the state $x(t)$, meet the condition $0 \leqslant \tau(t) \leqslant \tau < \infty$ and $\dot{\tau}(t) \leqslant h < \infty$. In [15,16], the author propose a new mathematical model with multiple additive time-delays, which relates to the practical situation in networked control system has the following form:

$$\dot{x}(t) = Ax(t) + A_h x\left[t - \sum_{i=1}^{n} \tau_k(t)\right],$$

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where $0 \leqslant \tau_i(t) \leqslant \tau_i < \infty, \dot{\tau}_i(t) \leqslant h_i < \infty$. This model contains multiple delay components in the state. [17–20] analysis the stability of systems with two successive delay components. which is

$$\dot{x}(t) = Ax(t) + A_h x(t - \tau_1(t) - \tau_2(t))$$

and some stability criteria are established. However, when constructing the Lyapunov–Krasovskii functional, approaches in [17–26] do not make full use of the information about $\tau(t)$, $\tau_1(t)$ and $\tau_2(t)$, which would be inevitably conservative to some extent. What is more, the purpose of reducing conservatism is still limited due to the existence of multiple coefficients the number of LMIs decision variables, from a theoretical point of view, still remains challenging.

Motivated by the above discussions, in this paper, the problem of improved delay-dependent stability criteria for continuous system with two additive time-varying delay components has been studied. By exploiting a new Lyapunov–Krasovskii functional, obtain the identical maximum allowable delay bounds while depending on much less decision variables, we derived a new and less conservative delay-dependent stability condition for system with two additive delay components. Finally, an illustrative example is provided to demonstrate the effectiveness and less conservativeness of the proposed methods.

Notations: Throughout this paper, $\Re^n$ denotes the n-dimensional Euclidean space and $\Re^{n\times m}$ refers to the set of all $n \times m$ real matrices. $*$ represents the elements below the main diagonal of a symmetric matrix. The superscripts T and -1 stand for matrix transposition and matrix inverse, respectively; $x_t := x(t+\theta), \theta \in [-\tau, 0], t \geqslant 0$ denotes by n-dimensional real vector-valued continuous function $x(t), t \in [-\tau, \infty)$.

2. Preliminaries

In this paper, we consider the following continuous system with two additive time-varying delay components described by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_h x(t - \tau_1(t) - \tau_2(t)), \\ x(t) &= \phi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathsf{T}} \in \mathcal{R}^n$ is the state of the system. A and A_h are known system matrices of appropriate dimensions corresponding to non-delayed and delayed, respectively. $\phi(t)$ is the initial condition on the segment $[-\tau, 0]$. $\tau_1(t)$ and $\tau_2(t)$ represent the two time-varying delay components in the state, and satisfying

$$0 \leqslant \tau_1(t) \leqslant \tau_1 \leqslant \infty, \quad \dot{\tau}_1(t) \leqslant h_1 \leqslant \infty, \quad 0 \leqslant \tau_2(t) \leqslant \tau_2 \leqslant \infty, \quad \dot{\tau}_2(t) \leqslant h_2 \leqslant \infty. \tag{2}$$

where τ_1, τ_2, h_1 and h_2 are constants.

We denote

$$\tau(t) = \tau_1(t) + \tau_2(t), \quad \tau = \tau_1 + \tau_2, \quad h = h_1 + h_2. \tag{3}$$

$$\chi(t) = [x^{\mathsf{T}}(t),\ x^{\mathsf{T}}(t - \tau_1(t)),\ x^{\mathsf{T}}(t - \tau(t)),\ x^{\mathsf{T}}(t - \tau_1),\ x^{\mathsf{T}}(t - \tau),\ x^{\mathsf{T}}(t - \tau_2(t)),\ x^{\mathsf{T}}(t - \tau_2)]^{\mathsf{T}}, \tag{4}$$

$$\Im_i = [\underbrace{0, \ldots, 0}_{i-1}, I, \underbrace{0, \ldots, 0}_{7-i}]. \tag{5}$$

such that system (1) can be written as

$$\dot{x}(t) = Ax(t) + A_h x(t - \tau(t)) = (A\Im_1 + A_h \Im_3)\chi(t). \tag{6}$$

First of all, we will give some lemma about system (6), which plays an important role in the derivation of our result.

Lemma 2.1. *For any constant matrices $X \in \mathbb{R}^{n\times n}$ and $Y \in \mathbb{R}^{n\times n}, \Omega = \begin{bmatrix} R & X^{\mathsf{T}} \\ X & R \end{bmatrix}$, scalars $0 \leqslant \tau_0 \leqslant \tau(t) \leqslant \tau_M$, and vector function $\dot{x} : [-\tau_M, -\tau_0] \to \mathbb{R}^n$, such that the following integrations are well defined, then*

$$-\int_{t-\tau_M}^{t-\tau_0} \dot{x}^{\mathsf{T}}(s) R \dot{x}(s) ds \leqslant -\underline{\chi}^{\mathsf{T}}(t) \begin{bmatrix} (e_3 - e_2)^{\mathsf{T}} \\ (e_2 - e_4)^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}} \Omega \begin{bmatrix} (e_3 - e_2)^{\mathsf{T}} \\ (e_2 - e_4)^{\mathsf{T}} \end{bmatrix} \underline{\chi}(t), \tag{7}$$

where

$$\underline{\chi}^{\mathsf{T}}(t) = [x^{\mathsf{T}}(t), x^{\mathsf{T}}(t - \tau(t)), x^{\mathsf{T}}(t - \tau_M), x^{\mathsf{T}}(t - \tau_0), \ldots, x^{\mathsf{T}}(t - \tau_m(t))], e_i = [\underbrace{0, \ldots, 0}_{i-1}, I, \underbrace{0, \ldots, 0}_{m+2-i}].$$

Proof. When $\tau_0 \leqslant \tau(t) \leqslant \tau_M$, together with Jessen inequality [1], we can obtain