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Sliding and oscillations in fisheries with on–off harvesting and different switching times

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ABSTRACT

In this paper, we propose a fishery model with a discontinuous on–off harvesting policy, based on a very simple and well known rule: stop fishing when the resource is too scarce, i.e. whenever fish biomass is lower than a given threshold. The dynamics of the one-dimensional continuous time model, represented by a discontinuous piecewise-smooth ordinary differential equation, converges to the Schaefer equilibrium or to the threshold through a sliding process. We also consider the model with discrete time impulsive on–off switching that shows oscillations around the threshold value. Finally, a discrete-time version of the model is considered, where on–off harvesting switchings are decided with the same discrete time scale of non overlapping reproduction seasons of the harvested fish species. In this case the border collision bifurcations leading to the creations and destruction of periodic oscillations of the fish biomass are studied.

1. Introduction

The exploitation of unregulated open access resources, such as fisheries, is characterized by a typical prisoner dilemma, often denoted as the 'tragedy of the commons' after [1]. In fact, free entry and individual profit maximization eventually lead the stock to very low levels, such as the *open access* equilibrium (see [2]). Thus, the main consequences are overfishing and fleets with overcapacity, leading to biological and economic inefficiencies, i.e. low levels of both fish and profits. Therefore, central institutions (fisheries management agencies) usually enforce various forms of regulation, ranging from setting harvesting restrictions (limiting the level of fishing effort, setting a total annual catch quota (TAC), establishing vessels buy-back programs, etc.), to imposing taxes on catches or limiting the kinds of species to be caught or the regions where exploitation is allowed (see e.g. [2–4]).

Over the last 50 years advanced mathematical bioeconomic models have been proposed to regulate the sustainable management of fisheries, assuming that central authorities solve optimal control problems to take into account biological, economic and social constraints. In practice, the real management of fisheries is achieved by trying to stabilize the biomass around a target level B, typically the *Maximum Sustainable Yield* (MSY). In 2002, during the Earth Summit, the EU member states committed themselves to "maintain or restore stocks to levels that produce the MSY with the aim of achieving these goals for depleted stocks on an urgent basis and where possible not later than 2015".[1] In other cases, the target level has been identified as the more conservative *Maximum Economic Yield* (MEY), as reported in [5]. The estimate of a target level is usually

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[1] See the Johannesburg Plan of Implementation, article 31a at http://www.johannesburgsummit.org/html/documents/summit_docs/2309_planfinal.htm, last accessed on December the 28th, 2012.

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based on a mathematical model for the single population under consideration (for instance employing the Schaefer model [6]) and depends on the regulator's goals and preferences.

This idea of stabilizing the stock around a target level is supported by the theory. In fact, as established by Clark and Munro in [7], if a regulator tries to maximize the present value of economic benefits derived from fishing, the resulting optimal control is a *bang-bang* policy to steer the system at the desired target: harvesting effort should be at a maximum level if the resource is above the target level, whereas no effort must be exerted if the resource is below the target (see Appendix A for details). However, the target level is greatly influenced by economic parameters (especially by the regulator's time preference).[2] To make things more complicated, these resources are often shared among several countries (for instance in the case of transboundary stocks) and so the definition of a proper target is taken by several decision makers. Once the kind of target is decided, it is extremely difficult to correctly assess it for single fish populations, as stock dynamics depend on the whole ecosystem and on environmental factors as well (see on this point [8]). Moreover, a bang-bang rule is hardly accepted by fishermen, especially if they have overcapacity and because of the strategic interaction among exploiters (the 'tragedy of the commons'). In any case there is the problem of preventing bycatches, which endanger the whole ecosystem. Not surprisingly, most fisheries managed to achieve the MSY level are overexploited.

Simple adaptive and self-regulating rules of thumbs are quite often employed in fishery management. One such rule is the so called 'Management by reference points', commonly applied in many North American fisheries (see [9] for an overview of the method and its principal drawbacks). For instance, the 40–10 harvest control rule (or the analogous 25–5 rule) of the Pacific Fisheries Management Council imposes constant harvesting when biomass is above 40% of the virgin stock size; when biomass is between 10% and 40% fishing effort must be progressively reduced and any harvesting activity must be stopped when biomass is below 10%.[3]

In this paper we address a simplified version of this rule, suggested by Clark in [10] to mimic real-world fishery management. This rule states that harvesting effort E can be proportional to fish biomass if the latter is above a threshold $B > 0$; however, total catch must be drastically reduced (for example to zero) whenever biomass falls below the threshold B (see also [11,12]). Considering the severe overexploitation in fisheries, this model can be considered as a good approximation of a management by reference points (given that the highest reference point is hardly crossed). Clark suggests that this threshold typically is less than or equal to the regulator's target biomass level (the MSY, the MEY or another target, see Appendix A for details). This threshold policy clearly introduces a discontinuity in the harvesting function, so that the dynamic model that represents this kind of fishery becomes piecewise continuous, as the threshold B separates the state space into two adjacent regions, the one where harvesting is allowed from that where it is forbidden (or strongly reduced).

The aim of this paper is to explore the main consequences of this specific on–off harvesting control for the resource dynamics and to show the route of complexity in the model because of this apparently simple rule. In particular, we make various assumptions on the time scale of natural growth and harvesting as well as on the time when the regulator assesses if the threshold B has been crossed or not. In this way, we can study various specifications of the model, to carry out an analysis which is more consistent with the real problem at hand. In the case of a continuous time scale, this kind of dynamical system may give rise to the so-called *sliding motion-stabilization* by means of a very rapid switching between the application and the interruption of the harvesting activity, leading to a fast convergence towards the threshold B. However, even if the long run behavior of such a system may appear to be a good one, as it is fully controllable by the regulator, it is quite unrealistic because it implies that an infinity of actions takes place in a finite amount of time, a property denoted as "Zeno" (see e.g. [13]); hence it is violated the fundamental requirement that alternating decisions of activating and suppressing harvesting cannot be infinitely fast. Indeed, harvesting decisions cannot be continuously revised, and are taken at discrete times, with a minimum time interval, say Δs, between two successive switchings. This leads to the second model under consideration, which is an hybrid dynamical system, as growth of fish species is modeled in continuous time whereas policy decisions occur at discrete time pulses, and causes oscillations around the threshold value B, with amplitude that depends on effort jump ΔE and switching time Δs. The study of hybrid and piecewise continuous dynamical systems is not easy, and is mainly performed numerically.

The third formulation of the model is in discrete time. As a matter of fact, population dynamics models are often formulated in such a way (see for instance [14,15]). Therefore, if we assume discrete time growth of fish population, i.e. seasonal and non overlapping birth rates, together with harvesting activities and control with the same discrete time scale, the dynamics of the system can be modeled through the iteration of a discontinuous one-dimensional map. For this model, we obtain some analytical results and characterize in a more complete way the periodicity that arises from the above mentioned threshold policy. In this case the creation of periodic cycles around B can be fully characterized by the study of border collision bifurcations (see e.g. [16–18]) and periodicity regions in the space of control parameters B and ΔE can be obtained.

So, the model proposed in this paper to analyze a very simple (even if frequently used in practice) on–off fishery regulatory policy gives us the opportunity to explore some interesting mathematical properties of discontinuous dynamical systems under different time scales, from continuous to discrete through the hybrid model as an intermediate case. This mathematical exercise allows us to explore analytically and numerically some dynamical properties of the model and to

[2] As reported in [8], another paradox in the management of the resource is that an equilibrium which involves the extinction of a resource could be regarded as 'optimal' provided that the regulator discounts very heavily future wealth.

[3] See for details the Pacific Fishery Management Council, proposed harvest specifications management measures for the 2011–2012 Pacific Coast Groundfish Fishery at http://www.pcouncil.org/wp-content/uploads/1112GF_SpexFEIS_100806-FINAL_feb21_.pdf, last accessed on December the 28th, 2012.