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An alternative linearization approach applicable to hysteretic systems

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ABSTRACT

In this paper a method is proposed for equivalent linearization of nonlinear restoring forces being governed by differential equations in weakly nonlinear systems. These types of restoring forces cannot be linearized by employing conventional approximate approaches. Two analytical examples are used to show the accuracy of the proposed method. The application of the method to hysteretic systems is examined by constructing equivalent linear representation for Bouc–Wen model in its general formulation. Numerical investigations reveal that the proposed method is efficient in dynamic behavior analysis of weakly nonlinear hysteretic systems.

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1. Introduction

Nonlinearities in engineering structures introduce difficulties in modeling, analysis and identification of their corresponding mathematical models. In modeling, finding appropriate models capable of capturing the actual nonlinear mechanisms has remained a challenging task in structural dynamics. Even with a suitable nonlinear model being available, the equations governing the dynamic response of a structure- which are a set of nonlinear differential equations- cannot be solved analytically. Therefore, approximate or numerical solutions are often used in order to obtain the structural dynamic responses under certain excitation conditions. It is worth mentioning that the type of nonlinearity and external excitation forces greatly affect the availability and accuracy of these solutions. By accurate modeling and appropriate analysis, identification of the nonlinearities is straightforward.

The mathematical theories for analysis and identification of linear structures have been developed well and many established strategies have been proposed. These theories allow one to understand all possible linear structural behavior. There are not such general theoretical approaches existing for treating non-linear structures. Among many methods proposed for dealing with a nonlinear structure, equivalent linearization has been used the most. That is because by using an equivalent linear model, it is possible to apply the approaches available for analyzing linear structures to nonlinear structures too.

Many linearization techniques can be found in the literature. The pioneer in this subject area is Caughey [1–3] who proposed the replacement of a nonlinear system by a linear one with the same excitation. He obtained the equivalent linear system by minimizing the mean-square error between two systems in a statistical sense. Following Caughey, Iwan [4] proposed to construct the equivalent linear model for a nonlinear structure through minimization of the average of the differences between the two systems. One of the approximation techniques mostly used for nonlinear structures under random excitation is the stochastic linearization (SL) [5,6]. The Bouc–Wen model has been used in conjunction with the statistical linearization method to approximate the response statistics of a variety of stochastically excited hysteretic oscillators

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[7,8]. Hurtado and Barbat [9] studied the sources of errors when conventional statistical linearization is applied to Bouc–Wen model. They then proposed a new technique based on a combination of Dirac and Gauss densities to obtain a more accurate equivalent linear representation for the Bouc–Wen model. Fujimura and Der Kiureghian [10] extended the method pioneered by Caughey [1–3] for linearization of nonlinear systems under random excitation. In their method the tail probability of the linear response is compared with a first-order approximation of the tail probability of the nonlinear response and the equivalent linear model is obtained. Giaralis and Spanos [11] employed the statistical linearization approach and developed a method for determining effective natural frequency and damping coefficient for nonlinear systems.

Another linearization approach is the harmonic balance method [12,13]. In this method, which is applicable to the systems with harmonic excitation, the response is considered as a linear combination of sub and supper harmonies. The coefficients are then calculated by balancing the energy of different harmonies between excitation force and the response.

The describing function method as a linearization method, which is mostly used in nonlinear control engineering, is an extension of the method proposed by Krylov and Bogoliubov [12]. In this method each nonlinear element is replaced with a quasi-linear descriptor. The descriptor approximates the nonlinear system by a linear transfer function having a gain dependent upon the input amplitude. This is in contrast with a linear system where its transfer function is independent of the input amplitude. In a sense, the describing function method is the harmonic balance method when one fundamental harmonic is considered in the analysis.

The above described methods have been applied by many researchers for analysis and identification of nonlinear structures. A comprehensive literature survey on this subject can be found in [14]. The harmonic balance and describing function methods have seldom been applied to nonlinear systems consisting of elements having restoring forces being governed by differential equations, e.g. hysteretic systems. In this paper the difficulties arising when these methods are applied to such nonlinear systems are described.

Periodic response analysis of hysteretic systems has been investigated by many researchers in the past. Caughey [15] used the averaging method and obtained the approximate solution of a bilinear hysteretic single degree of freedom system subjected to harmonic excitation. Capecchi and Vestroni [16,17] and Capecchi [18] employed the harmonic balance method and studied the periodic response and its stability of two classes of hysteretic systems: stiffness-degrading and stiffness-strength degrading systems. Bifurcation analysis of a hysteretic damping model composed of Maxwell's and Kelvin–Voigt's models was performed by Chang [19] by using the averaging method. Papers considering harmonic analysis of the hysteretic Bouc–Wen model in its general formulation are rare in the literature. Wong et al. [20] employed the Galerkin/Levenberg–Marquardt method and analyzed the multiharmonic steady-state response of a nonlinear system containing hysteretic Bouc–Wen model. They obtained the multi-valued resonant curves of the Bouc–Wen model and the corresponding subharmonic and super-harmonic respoces. The first order Bouc–Wen model, i.e. when $\bar{n} = 1$, has been the subject of analysis in many papers. Okuizumi and Kumara [21] studied the dynamic behavior of the Bouc–Wen model and the stability of its response by employing the method of multiple scales. They approximated the restoring force of the Bouc–Wen model by using a piecewise power series expression. The Krylov–Bogolyubov–Mitropolsky (KBM) method with harmonic balance method was adopted by Shen and Lin [22] to investigate the response of a weakly nonlinear vibratory system. They used the first order Bouc–Wen model in their analysis. The second order approximate solution and its stability of a hysteretic system subjected to parametric excitation were obtained by Okuizumi and Kumara [23]. Shen et al. [24] employed the first order Bouc–Wen model to represent the damping force of a magnetorheological (MR) damper. They obtained the equivalent linear properties of the Bouc–Wen model by adopting the equivalent linearization and averaging methods. They then identified Bouc–Wen model parameters by using experimental results.

The contribution of this paper is twofold: first, a method is proposed which can be used for obtaining equivalent linear representations of nonlinear restoring forces being governed by differential equations. These types of restoring forces cannot be treated by using conventional approximate methods such as harmonic balance or describing function methods. The proposed method considers that under harmonic excitation the response is harmonic. This assumption is applicable to weakly nonlinear systems. Second, harmonic equivalent linearization of the Bouc–Wen model in its general formulation is presented.

The remaining of this paper is organized as follows. In next two sections the problem statement and the linearization approach are presented. The method is validated by using two representative examples in section four. In section five application of the proposed method to hysteretic systems is examined and an equivalent linear representation for Bouc–Wen hysteretic model is obtained. Section six considers obtaining multiple solutions of the equivalent linear model. Finally conclusions are drawn, references and appendices are presented.

2. Problem statement

Consider finding the steady state response of the following weakly nonlinear vibratory system,

$$\begin{cases} E(x) + g(x,\dot{x}) = f\sin(\omega t) \\ \frac{\partial^n g(t)}{\partial t^n} = h\left(x(t), \frac{\partial x(t)}{\partial t}, \frac{\partial^2 x(t)}{\partial t^2}, \ldots, \frac{\partial^{\bar{m}} x(t)}{\partial t^{\bar{m}}}, \frac{\partial^{n-1} g(t)}{\partial t^{n-1}}, \frac{\partial^{n-2} g(t)}{\partial t^{n-2}}, \ldots, ng(t)\right) \end{cases} \quad (1)$$

where E is a liner differential operator,

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