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Passivity analysis of memristive neural networks with different memductance functions

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A B S T R A C T

Memristive neural networks have captured the attention of physicists, biologists, ecologists, economists and social scientists. In this paper, we formulate and investigate a class of memristive neural networks with two different types of memductance functions. Some succinct criteria in terms of linear matrix inequalities for the passivity are proposed. Meanwhile, based on the derived criteria, some stability criterion are obtained for the memristive neural networks. These theoretical analysis can characterize the fundamental electrical properties of memristive systems and provide convenience for applications.

1. Introduction

Recent developments in the simulation of different types of memristors have been rapidly translated to studies of memristive neural networks [1–10]. The memristive neural networks have features of complex brain networks-such as node degree, distribution and assortativity-both at the whole-brain scale of human neuroimaging. With the development of application as well as many integrated technologies, memristive neural networks have proven as a promising architecture in neuromorphic systems for the high-density, non-volatility, and unique memristive characteristic [1,3,5,6]. Taking a physical perspective, a memristive neural network is a hybrid nanomemristor/transistor logic circuit, where the artificial neurons are implemented by using nanomemristors/transistors. In this field, Pershin and Di Ventra [5] highlight an important finding-that the electronic (memristive) synapses and neurons can represent important functionalities of their biological counterparts. And so, of course, memristive neural networks of the neuromorphic computing architecture will enjoy great potentials in developing high performance parallel computing systems.

Consider the memristive neurodynamic system governed by the following equations: for $i = 1, 2, \ldots, n$,

$$\begin{aligned} \dot{x}_i(t) &= -x_i(t) + \sum_{j=1}^{n} w_{ij}(x_i(t))f_j(x_j(t)) + u_i(t), \\ y_i(t) &= f_i(x_i(t)), \end{aligned} \tag{1}$$

where $x_i(t)$ is the voltage of the capacitor $\mathbf{C}_i$, $u_i(t)$ and $y_i(t)$ denote external input and output, respectively, $f_j(\cdot)$ is the feedback function satisfying $f_j(0) = 0$, $w_{ij}(x_i(t))$ represents memristor-based weights, and

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$$w_{ij}(x_i(t)) = \frac{\mathbf{W}_{ij}}{\mathbf{C}_i} \times \mathrm{sgin}_{ij}, \quad \mathrm{sgin}_{ij} = \begin{cases} 1, & i \neq j, \\ -1, & i = j, \end{cases} \tag{2}$$

in which $\mathbf{W}_{ij}$ denotes the memductances of memristors $\mathbf{R}_{ij}$. And $\mathbf{R}_{ij}$ represents the memristor between the feedback function $f_i(x_i(t))$ and $x_i(t)$.

Combining with the physical structure of a memristor device, then one can see that

$$\mathbf{W}_{ij} = \frac{\mathrm{d}\mathbf{q}_{ij}}{\mathrm{d}\sigma_{ij}}, \tag{3}$$

where $\mathbf{q}_{ij}$ and σ_{ij} denote charge and magnetic flux corresponding to memristor $\mathbf{R}_{ij}$, respectively.

Many studies show that pinched hysteresis loops are the fingerprint of memristive devices. Under different pinched hysteresis loops, the evolutionary tendency or process of memristive systems evolves into different forms. It is generally known that the pinched hysteresis loop is due to the nonlinearity of memductance function. As two typical memductance functions, in this paper, we discuss the following two cases.

Case 1: The memductance function $\mathbf{W}_{ij}$ is given by:

$$\mathbf{W}_{ij} = \begin{cases} a_{ij}, & |\sigma_{ij}| < \ell_{ij}, \\ b_{ij}, & |\sigma_{ij}| > \ell_{ij}, \end{cases} \tag{4}$$

where a_{ij}, b_{ij} and $\ell_{ij} > 0$ are constants, $i, j = 1, 2, \ldots, n$.

Case 2: The memductance function $\mathbf{W}_{ij}$ is given by:

$$\mathbf{W}_{ij} = c_{ij} + 3d_{ij}\sigma_{ij}^2, \tag{5}$$

where c_{ij} and d_{ij} are constants, $i, j = 1, 2, \ldots, n$.

According to the features of memristor given in case 1 and case 2, then the following two cases can be happen.

Case 1′: In the case 1, then

$$w_{ij}(x_i(t)) = \begin{cases} \hat{w}_{ij}, & \mathrm{sgin}_{ij}\frac{\mathrm{d}f_j(x_j(t))}{\mathrm{d}t} - \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} \leqslant 0, \\ \check{w}_{ij}, & \mathrm{sgin}_{ij}\frac{\mathrm{d}f_j(x_j(t))}{\mathrm{d}t} - \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} > 0, \end{cases} \tag{6}$$

for $i, j = 1, 2, \ldots, n$, where $\hat{w}_{ij}$ and $\check{w}_{ij}$ are constants.

Case 2′: In the case 2, then

$$w_{ij}(x_i(t)) \text{ is a continuous function, and } \underline{\Lambda}_{ij} \leqslant w_{ij}(x_i(t)) \leqslant \overline{\Lambda}_{ij}, \tag{7}$$

for $i, j = 1, 2, \ldots, n$, where $\underline{\Lambda}_{ij}$ and $\overline{\Lambda}_{ij}$ are constants.

Obviously, the memristive neural network (1) with different memductance functions is a state-dependent switched system or a state-dependent continuous system, which is the generalization of those for conventional artificial neural networks.

On the other hand, the problem of passivity analysis has been a research hotspot in neural networks [11–33]. In fact, passivity is a very important issue in the study of neural systems. Passivity is part of a broader and a general theory of dissipativeness. The main point of passivity theory is that the passive properties of systems can keep the systems internally stable. The passivity performance is a extremely vital problem of neural networks, which has also been widely applied in many areas, such as systems analysis and integration [11–16,18–33], networked control [17].

It is worth pointing out that although the importance of passivity has been widely recognized, where some novel and significant results are obtained [11–33], no related results have been established for memristive neural networks. The study of passivity analysis of memristive neural networks is very hard and very few results have been reported until now, since a plethora of complex nonlinear behaviors appear even in a simple memristive system. The main purpose of this paper, therefore, is to shorten such a gap by making the attempt to deal with the passivity analysis problem for a class of memristive neural networks with two different types of memductance functions. Meanwhile, the theory discussion on passivity would help to design efficient memristor-based neuromorphic circuits and study other memristor-based complex systems. In brief, the main contributions of this paper are summarized as follows:

(1) A generic framework of memristive neural networks with two different types of memductance functions is presented and systematic investigation on passivity analysis has been demonstrated.
(2) By using the same method, we also acquire the stability criterion for the memristive neural networks, without any repeated calculation. Clearly, the method has more advantages.
(3) The proposed method may be applied for analyzing other classes of neural networks or some other complex nonlinear systems. These issues will be the topics of future research.

The rest of this paper is organized as follows. Section 2 presents some preliminaries. Main results are derived in Section 3. Numerical examples are given to illustrate the effectiveness of the proposed method in Section 4. Finally, concluding remarks will be drawn in Section 5.