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Identification of constitutive parameters for fractional viscoelasticity

Zhao Xiao, Yang Haitian *, He Yiqian

State Key Lab of Structural Analysis for Industrial Equipment, Dept. of Engineering Mechanics, Dalian University of Technology, 116024 Dalian, PR China

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ABSTRACT

This paper develops a numerical model to identify constitutive parameters in the fractional viscoelastic field. An explicit semi-analytical numerical model and a finite difference (FD) method based numerical model are derived for solving the direct homogenous and regionally inhomogeneous fractional viscoelastic problems, respectively. A continuous ant colony optimization (ACO) algorithm is employed to solve the inverse problem of identification. The feasibility of the proposed approach is illustrated via the numerical verification of a two-dimensional identification problem formulated by the fractional Kelvin–Voigt model, and the noisy data and regional inhomogeneity etc. are taken into account.

1. Introduction

The fractional derivative has become an extremely adequate tool to model mechanical properties of viscoelastic materials [1], because it is such an intimate descriptor of viscoelastic materials behavior that only a small number of parameters are enough to accurately represent a particular material [1–5]. An important problem connected with the fractional rheological models, as mentioned by Lewandowski [6], is the estimation of the model parameters from experimental data. As a matter of fact, there are a number of literatures related to this issue. However, these work seem to be mainly driven by determining parameters of simple fractional viscoelastic devices (such as viscoelastic dampers, dynamic vibration isolators etc. [6–13]), or by identifying fractional viscoelastic parameters of simple and homogeneous structures (such as beams [14]), instead of the parameters identification of a homogeneous/inhomogeneous fractional viscoelastic field. On the other hand, these work were mostly carried out in the frequency domain. When measurement data are limited and polluted, an error may arise in the integral transform from time domain to the frequency domain, as shown in the Section 4.

With the above consideration, this paper attempts to tackle with two issues, i.e.

1. The identification of constitutive parameters for fractional viscoelastic fields, instead of for simple devices or simple homogeneous structures.
2. The identification of fractional constitutive parameters in the time domain, instead of in the frequency domain.

In the Section 1, a Finite Element (FE) equation is derived for the direct problem, which can be solved by the Laplace transform or FD method for 2-D homogenous and regionally inhomogeneous fractional viscoelastic fields.

* Corresponding author. Tel.: +86 41184707942; fax: +86 41184708400.
E-mail address: Haitian@dlut.edu.cn (Y. Haitian).

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In Sections 2 and 3, the identification of fractional viscoelastic constitutive parameters is treated as an optimization problem that is solved by an ACO algorithm [15].

The ACO algorithm needs the solution provided in the Section 1, but does not need derivatives of solutions with respect to constitutive parameters. These derivatives are demanded for the gradient based algorithm, and seem unease to calculate accurately due to a difficulty caused by the Gamma function. Although the ACO algorithm has successfully been applied to solve various kinds of inverse problems [16–19], it seems first time to be used to solve inverse fractional viscoelastic problems.

In the Section 4, an identification problem of fractional viscoelastic constitutive parameters for a plate with a rectangular opening is investigated. The impacts of regional inhomogeneity, noisy data, and spatial arrangement of measurement points etc. on the solution are taken into account. The solutions obtained by the proposed approach are less sensitive to the noisy data given in this paper, and different spatial arrangements of measurement points seem no significant impact on the solution. Numerical results indicate that the proposed approach is available for the identification of constitutive parameters of homogeneous/regionally inhomogeneous fractional viscoelastic fields in the time domain.

2. The governing equations of 2-D direct fractional viscoelastic problem

The equilibrium equation of two-dimensional quasi-static problems is given by [20]

$$\mathbf{L}^T\boldsymbol{\sigma} + \mathbf{b} = 0 \quad \mathbf{x} \in V \tag{1}$$

The relationship between the strain and displacement is described by

$$\boldsymbol{\varepsilon}(t) = \mathbf{L}\mathbf{u}(t) \tag{2}$$

$$\mathbf{L} = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 & \frac{\partial}{\partial x_2} \\ 0 & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} \end{bmatrix}^T \tag{3}$$

where $\boldsymbol{\sigma}(t) = \{\sigma_{x_1}(t), \sigma_{x_2}(t), \sigma_{x_1x_2}(t)\}^T$ and $\boldsymbol{\varepsilon}(t) = \{\varepsilon_{x_1}(t), \varepsilon_{x_2}(t), \varepsilon_{x_1x_2}(t)\}^T$ stand for the vectors of stress and strain, respectively, $\mathbf{x} = \{x_1, x_2\}$ represents the vector of coordinates, $\mathbf{b}$ refers to the vector of the body force, and $\mathbf{u}(t) = \{u_{x_1}(t), u_{x_2}(t)\}^{\mathrm{T}}$ represents the vector of displacements.

The boundary conditions are specified by

$$\mathbf{u}(t) = \mathbf{u}_S(t) \quad \mathbf{x} \in S_u \tag{4}$$

$$\mathbf{F}(t) = \mathbf{n}\boldsymbol{\sigma}(t) = \mathbf{F}_S(t)\mathbf{x} \in S_\sigma \tag{5}$$

where $\mathbf{n}$ denotes the vector of unit outside normal along the boundary, $\mathbf{u}_S(t)$ and $\mathbf{F}_S(t)$ are prescribed functions, the subscripts u and σ refer to the 'displacement' and 'stress', respectively, and $S_u + S_\sigma = S = \partial V$ designates the whole boundary of the problem.

We assume that V consists of MT sub-domains, i.e. $V = \sum_{\kappa=1}^{MT} V_\kappa$, and the constitutive relationship in V_κ is characterized by [1,6]

$$\boldsymbol{\sigma}_\kappa(t) = (E_0^\kappa \mathbf{C}_\kappa + E_1^\kappa \mathbf{C}_\kappa D^{\alpha_\kappa})\boldsymbol{\varepsilon}_\kappa(t) = E_1^\kappa \mathbf{C}_\kappa \Psi_\kappa \boldsymbol{\varepsilon}_\kappa(t) \tag{6}$$

$$\mathbf{C}_\kappa = \frac{1}{1-\upsilon_\kappa^2}\begin{bmatrix} 1 & \upsilon_\kappa & 0 \\ \upsilon_\kappa & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon_\kappa}{2} \end{bmatrix} \tag{7}$$

$$\Psi_\kappa = \frac{E_0^\kappa}{E_1^\kappa} + D^{\alpha_\kappa} \tag{8}$$

$$\varphi_\kappa = \frac{E_0^\kappa}{E_1^\kappa} \tag{9}$$

where E_0^κ and E_1^κ are the static and asymptotic storage moduli, respectively, υ_κ refers to the Poisson ratio, and D^{α_κ} denotes a Caputo fractional derivative operator defined by [21]

$$D^{\alpha_\kappa}(z(t)) = \left[\frac{1}{\Gamma(1-\alpha_\kappa)}\int_0^t \frac{\partial(z(\tau))}{\partial\tau}(t-\tau)^{-\alpha_\kappa}d\tau\right] \quad 0 \leqslant t \leqslant Te, \quad 0 < \alpha_\kappa < 1 \tag{10}$$

where Te is the up-bound of time, $\Gamma(\bullet)$ refers to the Gamma function defined by [22]

$$\Gamma(z) = \int_0^\infty e^{-\chi}\chi^{z-1}d\chi \tag{11}$$