Commun Nonlinear Sci Numer Simulat 18 (2013) 2048–2065

Contents lists available at SciVerse ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Chaos control in passive walking dynamics of a compass-gait model

Hassène Gritli *, Nahla Khraief, Safya Belghith

SysCom Laboratory, National School of Engineers of Tunis, University of Tunis El-Manar, BP. 37, Le Belvédère, 1002 Tunis, Tunisia

ARTICLE INFO

Article history:
Received 13 April 2012
Received in revised form 26 July 2012
Accepted 2 December 2012
Available online 27 December 2012

Keywords:
Compass-gait model
Chaos
Hybrid limit cycle
Linearization
Constrained controlled Poincaré map
chaos control

ABSTRACT

The compass-gait walker is a two-degree-of-freedom biped that can walk passively and steadily down an incline without any actuation. The mathematical model of the walking dynamics is represented by an impulsive hybrid nonlinear model. It is capable of displaying cyclic motions and chaos. In this paper, we propose a new approach to controlling chaos cropped up from the passive dynamic walking of the compass-gait model. The proposed technique is to linearize the nonlinear model around a desired passive hybrid limit cycle. Then, we show that the nonlinear model is transformed to an impulsive hybrid linear model with a controlled jump. Basing on the linearized model, we derive an analytical expression of a constrained controlled Poincaré map. We present a method for the numerical simulation of this constrained map where bifurcation diagrams are plotted. Relying on these diagrams, we show that the linear model is fairly close to the nonlinear one. Using the linearized controlled Poincaré map, we design a state feedback controller in order to stabilize the fixed point of the Poincaré map. We show that this controller is very efficient for the control of chaos for the original nonlinear model.

1. Introduction

In the context of bipedal walking, the passive dynamic walking (PDW) is a walking method for which the biped robot does not require an exogenous source of energy but it uses the effect of gravity to move down an incline. There are three motivations behind the study and the design of PDW. Firstly, the passive biped robot has a mechanically self-stabilizing locomotion dynamics. Secondly, the PDW is expected to considerably increase energy efficiency of the bipedal locomotion. Thirdly, the PDW is investigated in order to obtain additional insights into the design principles of legged locomotion in nature. Many researches have been done on the development of walking bipeds based on the principle of PDW [1,2,4,5,9,36]. Research conducted under the PDW are primarily an analysis of its properties (dynamics, stability, limit cycles, etc.). The most significant works on the PDW are those of McGeer [2], Goswami et al. [3,4] and Garcia et al. [5], dealing with the walking problem of a passive compass-like biped robot on a sloping ground. The objective is to replicate cycles of passive locomotion, and to analyze generated cycles. The principle in turn is to use the effect of gravity as an action to reproduce a stable periodic walking from some initial configurations. In this same context, in order to obtain an in-depth understanding of the PDW, the impulsive hybrid nonlinear dynamics of the compass-gait model has been intensively studied until nowadays. As a result, the PDW has been always served as an alternative point of departure for the active dynamic walking (ADW) of biped robots. In this locomotion method, the biped robot uses some actuators at its joints allowing it to move on some surface. The ADW of biped robots refers to an approach that emphasizes the passive dynamics of the legs, and generally avoids the use of high-gain control. This aims mainly to increase energy consumption of the biped robot.

* Corresponding author.
E-mail address: grhass@yahoo.fr (H. Gritli).

1007-5704/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.cnsns.2012.12.009

Biped robots have been known so far to have an impulsive hybrid nonlinear dynamics [13–15,35]. Because of such impulsive hybrid feature of the gait model of biped robots, control process of dynamic walking is very difficult. In literature, several control architectures for legged walking robots have been elaborated based or not on the PDW. Despite the motivations of the use of the PDW, control procedure of biped robots, and particularly of the compass-gait biped robot, appears nowadays to be a challenging problem in robotic community. The central objective in the control problem of biped robots is to have always a stable periodic gait with maximum energetic efficiency [35]. The compass-gait biped robot is a two-dimensional impact mechanical system that is simple enough to be amendable to analysis, yet complex enough to display a wealth of interesting phenomena. It is known that the PDW of the compass robot exhibits periodic stable gaits with remarkably human-like motion as the biped robot goes down a sloped ground. Furthermore, it is known also that the passive compass-gait model exhibits symmetric and asymmetric gaits, bifurcations and chaos as some bifurcation parameter is varied [3–12,16–21,34]. The attractive phenomenon exhibited in the PDW of the compass robot is the scenario of period-doubling bifurcation route to chaos before falling down at some bifurcation parameter. We showed in [19,21] that a cyclic-fold bifurcation is generated in the PDW of the compass-gait model giving rise to a period-three route to chaos. Motivated by interesting demonstrations of the PDW, we conduct this paper on the design of a controller for the compass-gait model in order to control chaos exhibited in its PDW. The motivations behind trying to control chaos in the compass gait model are noted like so:

- The passive compass gait presents the first determinant of the human gait [38,4], and it is the simplest kinematics that may exhibits a passive bipedal walking gait.
- The compass-gait biped robot is the simplest biped and we can extend it to a more complicated biped like a torso-driven biped robot, or a biped robot with knees, with flat feet, etc. With suitable choice of the parameters, these biped robots can exhibit almost passive dynamic gaits and then chaos. Accordingly, control chaos will be needed.
- The control of chaos is to eliminate it from the passive dynamic walking patterns and hence to obtain a periodic stable walk that have something of a human walking look.

In literature, many methods of control of chaos in dynamical systems have been established [25]. However, for walking dynamics of biped robot, chaos control is not extensively treated and only few works are found. The OGY method [23] and the delayed feedback control (DFC) [24] are the major approaches to controlling or stabilizing chaos in walking dynamics of biped robot. These two approaches have been used for stabilizing a multiple period gait into a period-one gait. Sugimoto and Osuka [26,27] used the DFC to stabilize a passive gait and to generate an unstable period-one gait. Besides, Harata et al. [28] applied the DFC to parametric excitation walking to suppress bifurcation in a model of planar kneed biped robot with semicircular feet. Moreover, Suzuki and Furuta [29] used also the OGY method for the stabilization in order to enlarge the walkable range of the passive walking. Furthermore, Kurz and Stergiou [30] explored how hip joint actuation with damper can be used to control locomotive bifurcations and chaos in a PDW model. They referred in [31] to an artificial neural network that utilizes hip joint actuations to control bifurcations and chaos in such passive model.

The design of an effective and efficient control scheme of chaos is one of central focuses in the field of nonlinear science [22]. Chaos control is based on the richness of response of chaotic behavior. It is known so far that a chaotic attractor has a dense set of unstable periodic orbits (UPOs) and the system often visits the neighborhood of each one of them. It is well known also that there is infinite number of UPOs embedded in the chaotic attractor. UPOs have been rightly called by Cvitanovic [32,33] the "skeleton" of chaos. They play a central role in chaotic dynamics. Generally, chaos control is used to track a specific UPO embedded in a chaotic attractor. In the interest to the control of chaotic dynamics, Ott et al. [23] formulated the following two key ideas: (1) designing controller by the discrete system model based on linearization of the Poincaré map and (2) using the property of recurrence of the chaotic trajectories and applying the control action only at the instants when the trajectory returns to some neighborhood of the desired state or given orbit. This method is known as the OGY method and is based mainly on the linearization of the Poincaré map.

In this paper, we will use the formalism of the OGY method in order to control chaos in the PDW of the compass-gait model. However, the impulsive hybrid nonlinear dynamics complicate the task to derive an analytical expression of the Poincaré map. Then, our strategy is based first on the linearization of the impulsive hybrid nonlinear dynamics of the compass-gait model around a passive hybrid limit cycle (UPO). This leads to obtain an impulsive hybrid linear model with a controlled jump. Relying on the linearized model, we will derive a constrained controlled Poincaré map. We will give a method for the numerical simulation of the developed constrained map. This allows to plotting bifurcation diagrams of some descriptors of the passive gait. Through these bifurcation diagrams, we will show that the linearized model is nearly identical to the nonlinear one. Using the constrained Poincaré map, fixed point of an UPO will be easy identified. Its stability is investigated by means of the linearized Poincaré map. Relying on the linearized controlled Poincaré map, we will design a state feedback control basing on the formalism of LMI in order to stabilize an unstable fixed point, or systematically to stabilize an UPO. Then, we will apply the control law in the original nonlinear dynamics of the compass-gait model. We will show that the designed controller based on the linearization around a passive limit cycle ensure the stabilization of the UPO and so the control of chaos in the PDW of the compass-gait biped robot.

This paper is structured in eight sections. In Section 2, the impulsive hybrid nonlinear model of the dynamic walking of the compass-gait model is given. The passive walking patterns exhibited in the compass-gait model are given also in this section. Section 3 is dedicated for the development of a linearized model of the nonlinear one. In Section 4, we determine an analytical expression of a constrained controlled Poincaré map. The numerical simulation of this map is provided also