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Synchronous motion of two vertically excited planar elastic pendula

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ABSTRACT

The dynamics of two planar elastic pendula mounted on the horizontally excited platform have been studied. We give evidence that the pendula can exhibit synchronous oscillatory and rotation motion and show that stable in-phase and anti-phase synchronous states always co-exist. The complete bifurcational scenario leading from synchronous to asynchronous motion is shown. We argue that our results are robust as they exist in the wide range of the system parameters.

1. Introduction

The elastic pendulum is a simple mechanical system which comprises heavy mass suspended from a fixed point by a light spring which can stretch but not bend when moving in the gravitational field. The state of the system is given by three (spherical elastic pendulum) or two (planar elastic pendulum) coordinates of the mass, i.e. the system has three (spherical case) or two (planar case) degrees of freedom. The equations of motion are easy to write but, in general, impossible to solve analytically, even in the Hamiltonian case. The elastic pendulum exhibits a wide and surprising range of highly complex dynamic phenomena [1–16].

For small amplitudes perturbation techniques can be applied, the system is integrable and approximate analytical solutions can be found. The first known study of the elastic pendulum was made by Vitt and Gorelik [17]. They considered small oscillations of the planar pendulum and identified the linear normal modes of two distinct types, vertical or springing oscillations in which the elasticity is the restoring force and quasi-horizontal swinging oscillations in which the system acts like a pendulum. When the frequency of the springing and swinging modes are in the ratio 2:1, an interesting non-linear phenomenon occurs, in which the energy is transferred periodically back and forth between the springing and swinging motions [1–6]. The most detailed treatment of small amplitude oscillations of both plane and spherical elastic pendula is presented in the works of Lynch and his collaborators [7,10–12]. For large finite amplitudes the system exhibits different dynamical bifurcations and can show chaotic behaviour [8,9,13–16].

The dynamics of elastic pendulum attached to linear forced oscillator has been studied by Sado [18]. She has shown a one parameter bifurcation diagrams showing different behaviour of the systems (periodic, quasiperiodic and chaotic). According to our knowledge this is the only study of considered systems, but one can find a lot of papers concerning dynamics of classical pendulum attached to linear or non-linear oscillator. Hatwal et al. [19–21] gives approximate solutions in the primary

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parametric instability zone, which allows calculation of the separate regions of periodic solutions. Further analysis enables us to understand the dynamics around primary and secondary resonances [22–26]. Then the analysis was extended to systems with non-linear base where non-linearity is usually introduced by changing the linear spring into nonlinear one [27–30] or magnetorheological damper [31]. Recently the complete bifurcation diagram of oscillating and rotating solutions has been presented [32]. Dynamics of two coupled single-well Duffing oscillators forced by the common signal has been investigated in our previous papers [33,34]. We have shown the detailed analysis of synchronization phenomena and compare different methods of synchronization detection.

In this paper we study the dynamics of two planar elastic pendula mounted on the horizontally excited platform. Our aim is to identify the possible synchronous states of two pendula. We give evidence that the pendula can synchronize both in the oscillatory and rotational motion. Moreover in-phase and anti-phase synchronizations co-exist. Our calculations have been performed using software Auto-07p [35] developed for numerical continuation of the periodic solutions and verified by the direct integration of the equations of motion. We argue that our results are robust as they exist in the wide range of the system parameters.

The paper is organized as follows. Section 2 describes the considered model. We derive the equations of motion and identify the possible synchronization states. In Section 3 we study the stability of different types of synchronous motion. Finally Section 4 summarizes our results.

2. Model of the system

The analysed system is shown in Fig. 1. It consists of two identical elastic pendula of length l_0, spring stiffness k_2 and masses m, which are suspended on the oscillator. The oscillator consists of a bar, suspended on linear spring with stiffness k_1 and linear viscous damper with damping coefficient c_1. The system has five degrees of freedom. Mass M is constrained to move only in vertical direction and thus is described by the coordinate y. The motion of the first pendulum is described by angular displacement φ and its mass by coordinate x_2, that represent the elongation of the elastic pendulum. Similarly the second pendulum is described by angular displacement ϕ and its mass by coordinate x_3. Both pendula are damped by torques with identical damping coefficient c_2, that depend on their angular velocities (not shown in Fig. 1). The small damping, with damping coefficient c_3 is also taken for pendula masses. The system is forced parametrically by vertically applied force $F(t) = F_0 \cos \nu t$, acting on the bar of mass M, that connects the pendula. Force F_0 denotes the amplitude of excitation and ν the excitation frequency.

The equations of motion can be derived using Lagrange equations of the second type. The kinetic energy T, potential energy V and Rayleigh dissipation D are given respectively by:

$$T = \frac{1}{2}(M+2m)\dot{y}^2 + \frac{1}{2}m\dot{x}_3^2 + \frac{1}{2}m(l_0 + y_{wst2} + x_3)^2\dot{\phi}^2 + m\dot{y}\dot{x}_3\cos\phi - m\dot{y}\dot{\phi}(l_0 + y_{wst2} + x_3)\sin\phi + \frac{1}{2}m\dot{x}_2^2$$
$$+ \frac{1}{2}m(l_0 + y_{wst2} + x_2)^2\dot{\varphi}^2 + m\dot{y}\dot{x}_2\cos\varphi - m\dot{y}\dot{\varphi}(l_0 + y_{wst2} + x_2)\sin\varphi \tag{1}$$

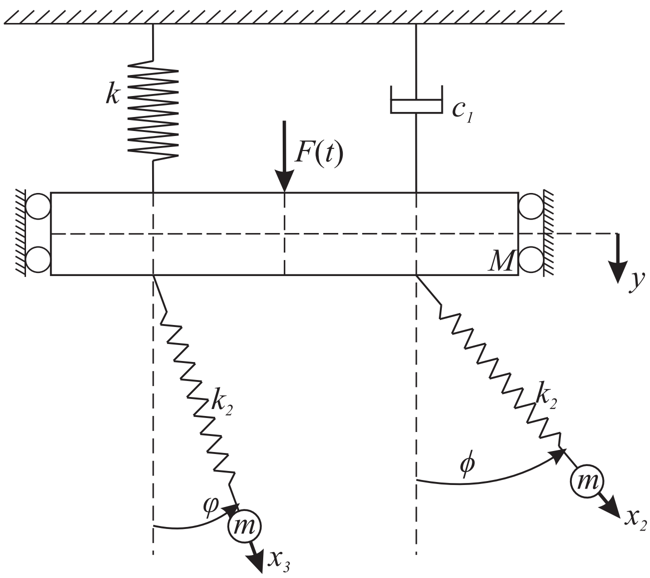

Fig. 1. Model of the system.