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Optimal estimation of parameters and states in stochastic time-varying systems with time delay

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ABSTRACT

In this study estimation of parameters and states in stochastic linear and nonlinear delay differential systems with time-varying coefficients and constant delay is explored. The approach consists of first employing a continuous time approximation to approximate the stochastic delay differential equation with a set of stochastic ordinary differential equations. Then the problem of parameter estimation in the resulting stochastic differential system is represented as an optimal filtering problem using a state augmentation technique. By adapting the extended Kalman–Bucy filter to the resulting system, the unknown parameters of the time-delayed system are estimated from noise-corrupted, possibly incomplete measurements of the states.

1. Introduction

Time-delayed dynamical systems, due to the vast variety of fields in science and engineering they can be relevant to, have attracted an increasing interest during past few decades. Some among their many practical applications are in areas such as manufacturing processes, robotics, neural networks, secure communication, traffic control, economics and biology [1–8]. For example cellular neural networks with delay coordinates may be considered as ideal models for neural networks which act using past knowledge, and various parameters in traffic control theory are also expressed in terms of delay coordinates. The primary complexity that introduction of the time-delay adds to the system is the growth of the phase space from a finite dimension to an infinite dimension. However, it has been shown [9] that the behavior of the infinite-dimensional delayed system can be predicted using a discretized finite dimension phase space. This approach transforms the original delay differential equations into either a large-dimensional dynamic map that maps the system behavior over a single maximal time delay, or else into a large system of ordinary differential equations.

There has been little work reported in the literature concerning parameter identification for time delay systems. Among those reported, most are limited to an identifiability analysis [10–13]. In 2002, Orlov et al. [14] developed an adaptive parameter identifier for linear dynamic systems with finitely many lumped delays in the state vector and control input to simultaneously identify the system parameters and the delay. In 2005, Mann and Young [15] examined the experimental data with empirical Floquet theory and principle orthogonal decomposition to estimate the parameters of a linear time-periodic delayed system with discrete delay from a reduced order map of the system. Later in 2012, Torkamani et al. [16] extended this approach for parameter identification in linear time-periodic delayed systems with distributed delay. In 2009, Tang and Guan [17] studied the problem of estimating time delay and parameters of time-delayed first order scalar chaotic systems by first converting the problem into an optimization problem with a suitable objective function and applying a particle swarm optimization algorithm. Sun and Yang [18] in 2010 exploited chaos synchronization for parameter

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identification of chaotic delayed systems with varying time-delay through using an adaptive feedback controller based on the Razumikhin condition and the invariance principle of functional differential equations in the framework of Lyapunov–Krasovskii theory. In 2011, parameter estimation of nonlinear time-varying DDEs with constant delay from fully and partially available data has been studied by Deshmukh [19] wherein an ideal case of parameter linearity with no external random disturbance was considered. In addition the system was assumed to be bounded-input bounded-output stable. In that paper Chebyshev spectral collocation is used to convert the delayed dynamic system (model) into an algebraic system with unknown parameters, and subsequently a standard least-squares optimization is employed to find the solution for the unknown parameters. While in all of the aforementioned studies, the critical role of measurement noise and model uncertainties are ignored, Basin et al. [20] considered the problem of optimal joint filtering and parameter identification in a linear stochastic time delay system through designing an optimal finite-dimensional filter.

What is clear above all is that the problem of identifying the unknown parameters of a stochastic time delay system from the time series response is still open to novel efficient techniques. In the current study a novel approach for parameter identification of DDEs is proposed through exploiting continuous time approximation and optimal filtering. Specifically, this study explores stochastic estimation of parameters and states of stochastic linear and nonlinear delay differential equations (DDEs) having time-varying coefficients and constant delay from an incomplete measurement. The stochastic delay differential equation is first discretized with a set of ordinary differential equations (ODEs) using the Chebyshev spectral continuous time approximation (CSCTA). Then the problem of parameter estimation in the resulting stochastic ODE system is represented as an optimal filtering problem using a state augmentation technique. Finally, using both conventional and extended Kalman–Bucy filters the unknown parameters of a nonlinear DDE are estimated from a noise-corrupted, possibly incomplete measurement of the states.

The paper is organized as follows: Section 2 describes how the stochastic DDE parameter estimation problem can be expressed as an optimal filtering problem. In Section 3 it is shown how the continuous time approximation can be used for approximating linear and nonlinear DDEs by an equivalent set of ODEs. Derivations of the extended Kalman–Bucy filter as an optimal filter based on the properties of a minimum variance unbiased estimator is presented in Section 4. Finally, in Section 5, the approach is implemented on various examples including linear and nonlinear, first and second order, scalar and higher-dimensional, time-delayed systems with single and multiple delays as well as incompletely measured states. Some advantages, restrictions, and concluding remarks regarding the proposed approach are presented in the conclusions.

2. Estimation problem in the form of optimal filtering

In a general stochastic parametric identification problem it is assumed that the form of the model is known only approximately due to imperfect knowledge of the dynamical model that describes the system and/or imperfect knowledge of parameters. The goal is to obtain the best estimation of the state as well as that of model parameters based on measured data that has a random component due to observation errors. In addition, a second source of stochastic excitation typically appears in the state dynamics as so-called process noise which can be either additive or multiplicative. Similarly, a stochastic parameter estimation problem in a delayed system is aimed to find the best estimate of the current (and consequently the delayed) state as well as the unknown parameters of a stochastically-excited delayed system model from measured data that contains a random observation error. The parameter estimation problem of a delayed system in its general form considered in this paper, can be formulated as an optimal continuous-time filtering problem in the form of a set of Ito stochastic delay differential equations as

$$\begin{aligned} d\mathbf{x}(t) &= \boldsymbol{m}(\mathbf{x}(t),\ \mathbf{x}(t-\tau),\ \mathbf{a}(t),\ t)dt + \boldsymbol{\sigma}(\mathbf{x}(t),\ t)d\boldsymbol{\beta}(t) \\ d\mathbf{z}(t) &= \boldsymbol{h}(\mathbf{x}(t),\ \mathbf{x}(t-\tau),\ t)dt + \mathbf{J}(t)d\boldsymbol{\eta}(t) \end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the current Ito process, $\mathbf{x}(t-\tau) \in \mathbb{R}^n$ is the delayed Ito process, $\mathbf{z}(t) \in \mathbb{R}^q$ is the measurement process, $\mathbf{a}(t) \in \mathbb{R}^r$ is a vector of unknown parameters, $\boldsymbol{m}()$ is the drift coefficient, $\boldsymbol{\sigma}()$ is the diffusion coefficient, $\boldsymbol{h}$ is the measurement model function, $\mathbf{J}$ is an arbitrary time-varying function independent of $\mathbf{x}$, and $\boldsymbol{\beta}(t)$ and $\boldsymbol{\eta}(t)$ are independent Brownian motion additive stochastic processes with $E[d\boldsymbol{\beta}(t)] = E[d\boldsymbol{\eta}(t)] = 0,\ E[d\boldsymbol{\beta}(t)d\boldsymbol{\beta}^T(t)] = \mathbf{Q}dt$ and $E[d\boldsymbol{\eta}(t)d\boldsymbol{\eta}^T(t)] = \mathbf{R}dt$ where $E[]$ represents the expectation operator. Note that in this paper the delayed system is considered to be excited by additive noise only. Therefore hereafter we will treat stochastic delayed differential equations with the diffusion coefficient only depending on t and not the Ito process. Under certain conditions [21], the filtering problem can also be formulated in terms of the stationary zero-mean Gaussian white noise processes formally defined as $\mathbf{v}(t) = d\boldsymbol{\beta}(t)/dt,\ \mathbf{w}(t) = d\boldsymbol{\eta}(t)/dt$ and differential measurement $\mathbf{y}(t) = d\mathbf{z}(t)/dt$. Therefore, the nonlinear stochastic delayed differential equation of Eq. (1) can be written as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \boldsymbol{f}(\mathbf{x}(t),\ \mathbf{x}(t-\tau),\ \mathbf{a}(t),\ t) + \mathbf{G}(t)\mathbf{v}(t) \\ \mathbf{y}(t) &= \boldsymbol{h}(\mathbf{x}(t),\ \mathbf{x}(t-\tau),\ t) + \mathbf{J}(t)\mathbf{w}(t) \end{aligned} \tag{2}$$

where $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are assumed to be both mutually independent and independent from the state and observation with constant covariance matrices of $\mathbf{Q}$ and $\mathbf{R}$, respectively, i.e. $\mathbf{v} \sim N(0, \mathbf{Q})$ and $\mathbf{w} \sim N(0, \mathbf{R})$. The stochastic term $\mathbf{v}(t)$ (the "process noise") can be considered to function as an approximation for the influence of the unknown dynamics of the process model. The time evolution of the states of the system and the unknown parameters of the stochastic model are