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Numerical analysis of dynamic crack propagation in biaxially strained rubber sheets

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ABSTRACT

This paper proposes a computational framework for dynamic crack propagation in rubber in which a nonlinear finite element analysis using cohesive zone modeling approach is used. A suddenly initiated crack at the center of biaxially stretched sheet problem is studied under plane stress conditions. A transient dynamic analysis using implicit time integration scheme is performed. In the constitutive modeling, the continuum is characterized by finite-viscoelasticity theory and coupled with the fracture processes using a cohesive zone model. This computational framework was introduced previously by the present authors (Elmukashfi and Kroon, 2012). In the current work, the use of a rate-dependent cohesive model is examined in addition to investigation of generalized biaxial loading cases. A Kelvin–Voigt element is used to describe the rate-dependent cohesive model wherein the spring is described by a bilinear law and dashpot with a constant viscosity is adopted. An explicit integration is used to incorporate the rate-dependent cohesive model in the finite element environment. A parametric study over the cohesive viscosity is performed and the steady crack propagation velocity is evaluated and compared with experimental data. It appears that the viscosity varies with the crack speed. Further, the total work of fracture is estimated using rate-independent cohesive law such that the strength of the cohesive zone is assumed to be constant and the separation work per unit area is determined form the experimental data. The results show that fracture-related processes, i.e. creation of new surfaces, cavitation and crystallization; contribute to the total work of fracture in a contradictory manner.

1. Introduction

Elastomers are important materials and widely used in many engineering applications, e.g., tires, springs, dampers, gaskets, bearings, oil seals, etc. Fracture mechanics in elastomers is of great importance in the design process and it is fundamental in some applications such as adhesion technology and elastomers wear. It has also been shown that many soft human tissues behave in a similar fashion as elastomers and their mechanical response can often be modeled in similar ways [34]. Accordingly, studying the crack growth problem in elastomers may shed some light on the dissection processes that cause severe damages of internal organs tissues.

Dynamic crack propagation concerns the onset of crack growth under dynamic (impact) loadings as well as dynamic (unstable) crack propagation in stressed solids. In such situations, the process is governed by the dynamic behavior of solids

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Nomenclature

a	crack length in deformed configuration
a_0	crack length in reference configuration
B_0	initial thickness of rubber sheet
$\mathbf{C}$	right Cauchy-Green deformation tensor
$\bar{\mathbf{C}}$	isochoric part of right Cauchy-Green deformation tensor
d	damage parameter in cohesive law
$\mathbf{e}_i$	basis vectors
$\mathbf{F}$	deformation gradient
$\bar{\mathbf{F}}$	isochoric part of deformation gradient
g_R	shear modulus in viscoelastic law
G_{Ic}	work of fracture
G_v	viscous contribution to work of fracture
h_{int}	height of rubber sheet in intermediate configuration
H_0	initial height of rubber sheet
$\mathbf{I}$	identity tensor
J	Jacobian
k_R	bulk modulus in viscoelastic law
$\mathbf{K}$	stiffness matrix for cohesive law
K_n	normal stiffness in cohesive law
l_{ce}	length of elements in cohesive zone
l_{cz}	length of cohesive zone
p	Lagrange multiplier
$\mathbf{S}_0$	instant second Piola–Kirchhoff stress tensor
t	iime
$\mathbf{T}$	traction vector in cohesive zone
$\mathbf{T}^e$	elastic part of traction vector in cohesive zone
$\mathbf{T}^v$	viscous part of traction vector in cohesive zone
T_n	normal component of traction vector in cohesive zone
$\mathbf{u}$	displacement vector
v_c	average crack velocity at steady-state
$\mathbf{v}_{tip}$	crack tip velocity vector
w_{int}	width of rubber sheet in intermediate configuration
W_0	initial width of rubber sheet
$\mathbf{x}$	Position vector in deformed configuration
$\mathbf{X}$	Position vector in reference configuration
$\mathbf{x}_{tip}$	crack tip position vector
α	damping coefficient in explicit time integration scheme
α_p	exponent in Ogden model
γ_s	surface energy in cohesive law
δ_c	surface separation at maximum stress in cohesive law
δ_f	surface separation at total failure in cohesive law
δ_n	normal component of displacement vector in cohesive zone
$\mathbf{\Delta}$	displacement vector in cohesive zone
Δa	crack growth in deformed configuration
Δa_0	crack growth in reference configuration
η	viscosity in cohesive law
ϕ	potential for cohesive law
$\boldsymbol{\kappa}$	internal state variables in cohesive law
μ	shear modulus
μ_p	shear modulus in Ogden model
λ_i	principal stretches
$\boldsymbol{\sigma}$	Cauchy stress tensor
σ_c	maximum stress in cohesive law
σ_{c0}	maximum elastic stress in cohesive law
σ_h	hydrostatic stress
$\boldsymbol{\tau}$	Kirchhoff stress tensor
$\boldsymbol{\tau}_0$	instant Kirchhoff stress tensor
τ	time translation
τ_i	time constant in viscoelastic law