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# Numerical evaluation of Paris-regime crack growth rate based on plastically dissipated energy

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ABSTRACT

The crack growth rate during cyclic loading is investigated via numerical simulations. The crack advancement is governed by a propagation criterion that relates the increment in plastically dissipated energy ahead of the crack tip to a critical value. Once this critical value is satisfied, crack propagation is modeled via a node release scheme. Thus, the crack growth rate is an output from the numerical simulation. The crack growth rate predicted by the proposed scheme is compared with published experimental crack growth data in the Paris-regime for selected metals. A good match is found between the experimentally observed crack growth rates and the numerically obtained results. The Paris coefficients are subsequently evaluated from the numerically obtained crack growth rates.



## 1. Introduction

Characterizing fatigue crack growth rate is an important part of safe life assessment of structural components. Obtaining the necessary data for use in life prediction tools requires detailed specimen preparation, pre-fatiguing the notch, fatigue crack growth rate measurements and interpretation of raw data, which are all costly and time consuming. There has been considerable efforts in developing methods to improve and optimize fatigue testing to limit the number of experimental tests and thereby reduce the overall cost without compromising accuracy. The avenues proposed for modeling fatigue crack growth and obtaining life prediction tools for crack propagation include among others, methods based on damage mechanics [1,2], stress intensity factors [3,4] and energy criteria [5–9].

Modeling of fatigue crack extension based on the critical plastic dissipation criterion dates back to the work of Rice [5] and has since been the topic of many analytical [6,10–12], experimental [13–16] and numerical investigations [7,9,17]. The dissipated energy criteria have been shown to be versatile for both microscopic and macroscopic analyses of fatigue [18]. A comparative assessment of the dissipated energy and other fatigue criteria can be found in Ref. [19]. Experimental results [16] have shown that the plastically dissipated energy can be used to determine crack propagation rates under both constant amplitude and variable amplitude cyclic loading.

The plastically dissipated energy can be directly linked to the accumulation of plastic strain. In metals, plastic strain is due to dislocation motion which is associated with fatigue [5,20]. An energy based approach to fatigue crack growth was developed by Weertman [11], considering a uniform distribution of edge dislocations at the crack tip. Based on the work

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**Nomenclature**

| | |
|---|---|
| $\Delta K$ | Stress intensity factor range |
| $da/dN$ | Fatigue crack propagation rate |
| $C$ | Paris coefficient |
| $m$ | Paris exponent |
| $\Delta a\|_N$ | Discrete equivalent of the continuous crack propagation rate |
| $W_e^p$ | Plastically dissipated energy in element |
| $W_{cr}^p$ | Critical plastically dissipated energy |
| $\overline{W}_{cr}^p$ | Critical plastically dissipated energy scaled with the element size |
| $h_e$ | Element size in the refined structured mesh |
| $D_k$ | Dissipation domain at $k$th iteration |
| $W$ | Width of compact tension specimen |
| $a_n$ | Initial crack length in the compact tension specimen |
| $\sigma_{max}$ | Maximum applied stress |
| $R$ | Load ratio ($P_{min}/P_{max}$) |
| $\sigma_y$ | Yield strength |
| $E$ | Elastic modulus |
| $\nu$ | Poisson's ratio |
| NRPZ | Number of elements in reverse plastic zone along crack path |
| $f_0$ | Overload ratio |
| $\sigma_{overload}$ | Maximum applied stress during the overload cycle |

of Bodner et al. [6], Klingbeil [7] proposed a technique for predicting fatigue crack growth in terms of the per-cycle rate of plastic energy dissipated in the reverse plastic zone formed at the crack tip upon a loading–unloading cycle. Klingbeil's approach is based on evaluating the plastically dissipated energy around a stationary crack under mode I loading using finite element analysis. The technique was later applied by Daily and Klingbeil [21] for stationary cracks under mixed mode loading conditions. Klingbeil's theory was recently extended and used by Smith [17] to examine the applicability of the dissipated energy criterion for predicting delayed retardation effects following a single tensile overload. A 3D boundary layer FE model was used to model a crack arbitrarily propagating at the rate of one element per cycle. Crack tip shielding effects were also accounted for. However, the results were found to be mesh dependent due to the arbitrary crack propagation rate. Moreover, the actual crack growth rate can only be determined via experimental calibrations in that scheme [17].

An alternative approach to predict fatigue crack propagation rate directly from FE simulations based on the plastically dissipated energy criterion was recently proposed by Cojocaru and Karlsson [9]. A two dimensional (2D) plane strain analysis was presented for fatigue crack growth rate changes due to negative load ratios and single and multiple tensile overloads. Qualitative agreement with experimentally observed rates was shown for these different load cases. In addition, the plastically dissipated energy criterion was also implemented for three dimensional modeling of fatigue crack growth to predict crack front profile changes (crack tunneling) under cyclic loading [22].

When subjected to fatigue loading, materials exhibit a linear portion on a log–log plot of fatigue crack propagation rate, $da/dN$, versus the applied stress intensity factor range, $\Delta K$. In the fracture mechanics approach to fatigue crack propagation, this linear portion of experimental data is described by the well known Paris' law [3,23]

$$\frac{da}{dN} = C\Delta K^m \tag{1}$$

where $C$ and $m$ are experimentally determined material properties commonly called Paris coefficients. In this work, we augment the approach proposed in Ref. [9] for modeling crack growth under cyclic stresses based on the plastically dissipated energy to numerically determine these Paris law coefficients. Comparisons with measured crack growth rate data is presented for a variety of ductile metals, including aluminum, titanium and nickel based alloys used in aerospace applications.

## 2. Numerical approach to predicting cyclic crack growth rate

### 2.1. Theory

In this work, the numerical scheme for predicting cyclic crack propagation rate is based on a continuum perspective [8,9]. Our premise is that fatigue cracks propagate due to cyclic material degradation in a process zone associated with the crack tip (see for example, Ref. [24]). The degradation of the material in the process zone is accompanied by significant plastic deformation if the material is ductile (due to dislocation motion in metals, and shear banding and crazing in polymers).