Engineering Fracture Mechanics 124–125 (2014) 196–216

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

A localized mapped damage model for orthotropic materials

Luca Pelà *, Miguel Cervera, Sergio Oller, Michele Chiumenti

CIMNE, International Center for Numerical Method in Engineering, Technical University of Catalonia (UPC), Campus Norte, Jordi Girona 1-3, 08034 Barcelona, Spain

ARTICLE INFO

Article history:
Received 16 September 2013
Received in revised form 16 March 2014
Accepted 25 April 2014
Available online 8 May 2014

Keywords:
Continuum Damage Mechanics
Orthotropy
Transformation tensor
Fracture
Crack-tracking
Masonry

ABSTRACT

This paper presents an implicit orthotropic model based on the Continuum Damage Mechanics isotropic models. A mapping relationship is established between the behaviour of the anisotropic material and that of an isotropic one. The proposed model is used to simulate the failure loci of common orthotropic materials, such as masonry, fibre-reinforced composites and wood. The damage model is combined with a crack-tracking technique to reproduce the propagation of localized cracks in the discrete FE problem. The proposed numerical model is used to simulate the mixed mode fracture in masonry members with different orientations of the brick layers.

1. Introduction

The mechanical behaviour of anisotropic materials involves properties that vary from point to point, due to composite or heterogeneous nature, type and arrangement of constituents, presence of different phases or material defects. A macroscopic continuum model aimed at the phenomenological description of anisotropic materials should account for (i) the elastic anisotropy, (ii) the strength anisotropy (or yield anisotropy, in case of ductile materials) and (iii) the brittleness (or softening) anisotropy [1].

Several materials can be considered, with an acceptable degree of approximation, to be orthotropic, even though some of them are not so in the whole range of behaviour. Modelling the elastic orthotropy does not present big difficulties, since it is possible to use the general elasticity theory [2]. On the other hand, the need to model the strength and nonlinear orthotropic behaviour requires the formulation of adequate constitutive laws, which can be based on such theories as plasticity or damage. In particular, although several failure functions have been proposed, the choice of a suitable orthotropic criterion still remains a complex task.

One of the more popular attempts to formulate orthotropic yield functions for metals in the field of plasticity theory is due to Hill [3,4], who succeeded in extending the von Mises [5] isotropic model to the orthotropic case. The main limitation of this theory is the impossibility of modelling materials that present a behaviour which not only depends on the second invariant of the stress tensor, i.e. the case of geomaterials or composite materials. On the other hand, Hoffman [6] and Tsai–Wu [7] orthotropic yield criteria are useful tools for the failure prediction of composite materials.

* Corresponding author. Address: Universitat Politècnica de Catalunya – BarcelonaTech, Department of Construction Engineering, C/Jordi Girona, 1-3 (Module C1 – Office 206-B), 08034 Barcelona, Spain. Tel.: +34 93 401 10 36; fax: +34 93 405 41 35.

E-mail addresses: luca.pela@upc.edu (L. Pelà), miguel.cervera@upc.edu (M. Cervera), sergio.oller@upc.edu (S. Oller), michele@cimne.upc.edu (M. Chiumenti).

http://dx.doi.org/10.1016/j.engfracmech.2014.04.027
0013-7944/

Nomenclature

$\mathbf{A}^{\sigma}$	stress transformation tensor
$\mathbf{A}^{\varepsilon}$	strain transformation tensor
$\mathbf{C}$	linear-elastic constitutive tensor
d	damage index
D	specific dissipated energy
E_i	Young's modulus referred to i-axes
f_{ii}	uniaxial strength in the i-th direction
f_{ij}	pure shear strength in the ij-th plane
F_{12}	interaction coefficient of Tsai–Wu criterion
$G_{f,i}$	mode I fracture energy per unit area along the i-th direction
G_{ij}	shear modulus in the ij-th plane
k_f	fibre volume fraction
K	parameter of Faria's criterion
n	parameter of Hankinson's formula
r	damage threshold internal variable
r_{ij}	direction cosines
x_i	coordinate system
$\boldsymbol{\varepsilon}$	strain tensor
θ	angle of orthotropy
$\boldsymbol{\Lambda}$	damage threshold surface shape tensor
υ_{ij}	Poisson's ratio in the ij-th plane
σ_i	i-th principal stress
$\boldsymbol{\sigma}$	stress tensor
$\bar{\boldsymbol{\sigma}}$	effective stress tensor
τ	equivalent stress
Φ	damage criterion function
ψ	free energy potential
ψ_0	elastic free energy potential
:	double contraction
$'$	apex denoting vectors/tensors defined in the principal axes of orthotropy
*	apex assigned to scalars/tensors defined in the mapped space
$\langle\cdot\rangle$	Macaulay brackets

Acronyms

CDM	Continuum Damage Mechanics
CMOD	crack mouth opening displacement
E-FEM	Elemental enrichment Finite Element Method
FE	finite element
FEM	Finite Element Method
FRP	fiber reinforced polymer
SCA	Smeared Crack Approach
X-FEM	eXtended Finite Element Method
2D	two-dimensional

For the description of incompressible plastic anisotropy, not only yield functions [8] and phenomenological plastic potentials [9] have been proposed over the years. Other formulation strategies have been developed, related to general transformations based on theory of tensor representation [10,11]. A particular case of this general theory, which is based on linearly transformed stress components, has received more attention. This special case is of practical importance because convex formulations can be easily developed and, thus, stability in numerical simulations is ensured. Linear transformations on the stress tensor were first introduced by Sobotka [12] and Boehler and Sawczuck [13]. For plane stress and orthotropic material symmetry, Barlat and Lian [14] combined the principal values of these transformed stress tensors with an isotropic yield function. Barlat et al. [15] applied this method to a full stress state and Karafillis and Boyce [16] generalized it as the so-called isotropic plasticity equivalent theory with a more general yield function and a linear transformation that can accommodate other material symmetries. Betten [17,18] introduced the concept of mapped stress tensor to express the behaviour of an anisotropic material by means of an equivalent isotropic solid (mapped isotropic problem). The same approach was later refined by Oller et al. [19–23] with the definition of transformation tensors to relate the stress and strain tensors of the orthotropic space to those of a mapped space, in which the isotropic criterion is defined. The stress and strain transformation tensors are symmetric and rank-four and establish a one-to-one mapping of the stress/strain components defined in one