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Improvement of some features of finite elements with embedded discontinuities

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ABSTRACT

This paper shows an analysis of the principal features of finite elements with embedded discontinuities. Particularly, two families of this kind of elements are analyzed, kinematically optimal symmetric and statically and kinematically optimal non-symmetric. The analyzed features are the variational formulation, damage model and how the discontinuity is introduced. A new definition of traction vector for symmetric family, dependent on the discontinuity length and the angle, is given. It is shown that kinematics and equilibrium are satisfied, without the problem of fictitious tractions as stated in the literature. To show the validity of this symmetric formulation, representative numerical examples illustrating the performance of the proposed formulation are presented.

1. Introduction

The idea of lumping a strain concentration into a line or surface for 2D and 3D quasi-static damage mechanics problems, respectively, has motivated the development and application of solid finite elements with embedded discontinuities (FEEDs) [8–14,27,30–32]. An extension of this technique was the development of finite elements simulating hinges in beams [5,15,16,20] and an extended formulation for the analysis of softening hinge lines in inelastic thick plates [6]. More recently, FEEDs have been used for dynamic fracture simulations [3,4,17]. In the formulation of these kinds of elements, there are mainly two requirements which must be satisfied in the localization zone: (1) equilibrium, traction continuity across the discontinuity interface and (2) kinematics, free relative rigid body motions of the two portions of an element split up by a discontinuity.

A comprehensive study of FEEDs is found in [18], where these elements are classified into three families: (1) statically optimal symmetric (SOS), which satisfies equilibrium but does not kinematics; (2) kinematically optimal symmetric (KOS), which satisfies kinematics and apparently does not satisfy equilibrium; and (3) statically and kinematically optimal non-symmetric (SKON), which satisfies both equilibrium and kinematics. New symmetric FEEDs, including mixed and assumed enhanced strain techniques, have been explored by [25], showing that although these symmetric FEEDs reduced the stress locking problem, the SKON formulations still provide better results.

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Nomenclature

$[\|\mathbf{u}\|]$	displacement jump
S	surface
Ω	domain
Γ	boundaries
$\mathbf{t}^*$	surface tractions
$\mathbf{u}^*$	prescribed displacements
H_S	Heaviside function
δ_S	Dirac delta function
Ψ	free energy density
$\boldsymbol{\varepsilon}$	strain tensor
$\boldsymbol{\sigma}$	stress tensor
$\mathbf{C}$	elastic tensor
d	damage variable
q	hardening/softening variable
H	hardening/softening parameter
γ	The damage multiplier
r_0	threshold value
φ	discrete free energy density
$\mathcal{T}$	traction vector
ω	discrete damage variable
$\dot{\bar{q}}$	hardening/softening variable
$\bar{\lambda}$	The damage multiplier
$\mathbf{C}^T$	continuum tangent constitutive operator
$\mathbf{C}_d^T$	discrete tangent constitutive operator
$\Psi(\boldsymbol{\varepsilon}^u)$	free energy density
$\Psi_S([\|\mathbf{u}\|])$	free discrete energy density,
$\bar{\boldsymbol{\varepsilon}}$	continuous strain field, is given by:
$\mathcal{T}_{x,y}$	traction in a global system
$\hat{\mathbf{u}}$	regular displacement field
$\mathbf{N}$	the standard vector of shape functions
$\mathbf{d}$	nodal displacement vector
$\mathbf{B}$	standard strain interpolation matrix, containing the derivatives of the standard shape functions $\partial(\mathbf{Nd}) = \mathbf{Bd}$

This paper provides an analysis of the principal features of finite elements with embedded discontinuities, particularly of the KOS and the SKON formulations. The equilibrium in the KOS formulation is satisfied by a new definition of traction as a function of the length of the discontinuity, in the sense that the two equations at the residual are forces.The SOS formulation was not included in this paper since its kinematics is not correct, contrary to the formulation of the strong discontinuity models where the good performance shown is attributed to its correct kinematics [7].

The outline of this paper is as follows. Section 2 presents the equations defining kinematics and the boundary value problem (BVP) of a solid with discontinuities. Also, this section provides the constitutive models to describe the behavior of the material in the continuum and a damage model for the development of discontinuities. Section 3 presents the energy functional of solids with strong discontinuities for the SKON and the KOS formulations. Section 4 shows the finite element approximation of the variational formulations with strong discontinuities. Numerical examples of elements with discontinuities which validate the proposed formulation are presented in Section 5. Finally, in Section 6, conclusions derived from the work and suggestions about future research are given.

2. Problem definition

2.1. Boundary value problem

Consider a 3D body, defined by an open bounded domain, $\Omega \in \mathbb{R}^3$, and boundary, Γ, (Fig. 1a), loaded until it undergoes a displacement discontinuity, $[|\mathbf{u}|]$, across a surface, S, where the inelastic deformations are concentrated. This discontinuity splits the domain into sub-domains such that, $\Omega = \Omega^- + \Omega^+ + S$, with two boundaries, $\Gamma = \Gamma^- + \Gamma^+$. The boundary conditions are the prescribed surface tractions, $\mathbf{t}^*$, on $\Gamma_\sigma = \Gamma_\sigma^- + \Gamma_\sigma^+$ and the prescribed displacements, $\mathbf{u}^*$, on $\Gamma_u = \Gamma_u^- + \Gamma_u^+$, such that $\Gamma_\sigma \cup \Gamma_u = \Gamma$ and $\Gamma_\sigma \cap \Gamma_u = \emptyset$. This problem may be idealized using two different approaches: Strong Discontinuity (SD)