Engineering Fracture Mechanics 117 (2014) 1–11

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Approximation of mode I crack-tip displacement fields by a gradient enhanced elasticity theory

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ARTICLE INFO

Article history:
Received 26 June 2013
Received in revised form 20 October 2013
Accepted 2 January 2014

Keywords:
Heterogeneous material
Gradient enhanced continuum
Crack mechanics
Asymptotic analysis
Deformation field

ABSTRACT

Gradient theories are capable of describing deformation of heterogeneous elastic materials better than classical elasticity theory since they are able to capture internal length effects. Here, crack-tip displacement fields at the tip of a mode I crack in gradient enhanced elastic materials are derived in closed form and contrasted with experiments. Heterogeneous materials, represented by discrete fiber networks, are analyzed in finite element models to judge the theory. It is shown that using a classical continuum approach to describe macroscopic singular-dominated deformation fields in heterogeneous materials lead to erroneous results because a structural effect that alters the displacement field becomes pronounced and results in severe blunting of crack-tips. A key conclusion is that the average segment length in the material gives the internal length scale parameter, used in the gradient enhanced continuum theory, hence allows for bridging between scales.

1. Introduction

This study addresses the effect of microstructure on the deformation behavior of strongly heterogeneous materials and especially its practical implications for understanding material failure in network materials such as nonwoven felts, paper, fiber composites, textiles and their alike. It has been reported that a sparse network structure (such as tissue) is not as sensitive to defects as dense network materials, Hägglund and Isaksson [1]. In a sparse network, a relatively large defect size is required to localize macroscopic fracture. A key question arises: why are sparse network structures relatively insensitive to flaws? There are several mechanisms that may lower the stress intensity at a macroscopic crack in a fiber material, such as plastic straining of the fibers or microscopic fiber fracture. An additional explanation, which is considered here, is a linearly elastic crack-tip blunting phenomenon caused by bending deformation of fiber segments in the vicinity of the crack-tip. The blunting effect may be observed visually: when loaded, blunted cracks have rectangular shapes rather than sharp elliptical as expected in linearly elastic fracture mechanics (LEFM), Fig. 1. Among others, Fleck and Qui [2] have reported that this effect is pronounced in e.g. Kagome lattices, which are a type of isotropic periodic planar lattices. To illustrate the blunting phenomenon, a digital video recorder was used to monitor the deformations at crack regions during fracture tests on specimens made of a sparse tissue paper material with approximately in-plane isotropic properties. Specimens were subjected to opening mode loading using a tensile testing machine (Bose ElectroForce, maximum force 400 N). Two initial crack lengths were prefabricated in specimens (2.7 mm and 3.2 mm) with a razor knife. The curves showing the evolution of the force with respect to the jaw opening displacement are given in Fig. 2. Up to the elongation of about 1.8 mm, the both curves are similar in spite of the initial crack length difference. For larger displacements, the deviating curves in Fig. 2 illustrate differences in the

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0013-7944/$ - see front matter
http://dx.doi.org/10.1016/j.engfracmech.2014.01.001

Nomenclature

x_1, x_2	Cartesian coordinates
r,θ	polar coordinates
δ_{ij}	Kronecker's delta
Δ	separation of the crack lips at $x_2 \to 0$
$\delta(a)$	crack lip separations at position $x_x = -a$
$u_i, \bar{u}_i$	displacement fields: LEFM, gradient enhanced
c	characteristic length
λ	length ratio r/c
L,l_s	fiber length, average segment length along fibers
K_I	mode I stress intensity factor
σ_{ij}, $\bar{\sigma}_{ij}$	local stress tensor, gradient enhanced stress tensor
$f_{ij}(\theta)$, $g_i(\theta)$	LEFM angular functions

damaging behavior of the specimens. For the specimen having the shorter initial crack length the continued crack growth failed to localize to the crack-tips while for the longer initial crack the continued crack growth were inclined to the tips. This behavior was observed in all ten experiments (five in each configuration) performed. Hence, the final failure does not seem to be sensitive to the presence of fairly small defects, an observation that confirms that low-density fiber networks are tolerant to the presence of small size defects, cf. [1].

Further, the visual results in the sequences shown in Figs. 2 and 3 suggest that there is a microstructural blunting effect of the tips; the shape of the crack is rectangular rather than sharp. The vertical length V of the crack in the specimen having the shorter slit did not evolve during loading (Fig. 3), while the horizontal length H of the crack varied slightly. At the beginning of the test it is hard to see due to the low resolution of images but H attains a low value of approximately 0.5 mm at an opening jaw displacement corresponding to the maximum loading force about 1.8 mm (see Fig. 2). Then H started to slightly decrease due to the load release induced by damage in the specimen. After the last picture in the sequence, the load was completely released whereupon the material returned close to its virgin state in the vicinity of the crack and the crack was closed, meaning that severe irreversible plastic deformations were not introduced during loading. For the specimen having longer prefabricated slit, crack-tip blunting was observed, i.e. H evolved up to final failure. This phenomenon eventually resulted in a large horizontal opening of the crack (after peak load). After complete unloading, the material located far away from the developed damage zone seen in Fig. 3, image 6, returned to its virgin state.

The crack-tip problem has over the years attracted much attention in the material science community and many models have been suggested to eliminate stress and strain singularities appearing in classical theory of elasticity. The early models of Barenblatt [3] and Dugdale [4], suggested more than half a century ago, assume a thin extension of a present crack over which cohesive stresses acts. These fairly simple models are based on the observed shapes of developed plastic zones at crack-tips in thin ductile metals. To obtain a more general mathematical theory, much work has been done in the field of nonlocal and gradient elasticity, which include length parameters in the constitutive equations that limit the magnitude of stress and strain. The physical motivation to introduce gradient theories was originally presented in the early 1960s by Toupin [5] and Kröner [6]. During that decade much work emerged in the field, e.g. Mindlin [7], Kröner [8], Mindlin and Eshel [9]. In the past four decades, numerous nonlocal and gradient theories incorporating material length scales have been suggested, cf. Eringen et al. [10–12], Triantafyllidis and Aifantis [13] or Aifantis [14]. Overviews of various gradient theories are now available in the literature, cf. [15]. Despite this increasing knowledge, fracture in network materials has traditionally

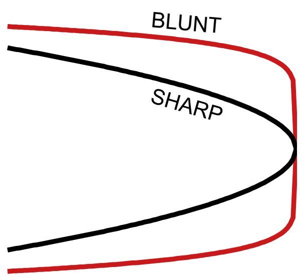

Fig. 1. Theoretical blunt and sharp crack-tips.