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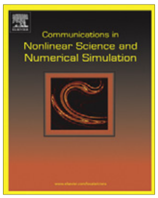

Determination of minimum required damping in stochastic following seas modeled by using Gaussian white noise

E. Üçer *, M. Söylemez

Istanbul Technical University, Faculty of Naval Architecture and Ocean Engineering, Maslak 34469, Istanbul, Turkey

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ABSTRACT

The aim of this study is to present an analytical method to determine the minimum required damping moment for a stable ship in stochastic following seas modeled by using Gaussian white noise. Stochastic differential equation is used as a mathematical model to represent rolling motion of a ship. First, the minimum required damping is obtained analytically by using Lyapunov function. Second, analytically obtained damping values are verified by integrating the nonlinear stochastic rolling motion equation by stochastic Euler method (Euler–Maruyama Schema) to deduce whether rolling motion is stable or not. It can be seen from the results of numerical computation that the ship is sufficiently stable for the minimum required damping value obtained by the use of Lyapunov function and the minimum required damping is highly dependent on natural frequency of roll, diffusion constant and maximum variation of initial metacentric height.

1. Introduction

Engineering problems in numerous areas such as the motion of ships, chemical reactions, etc., have been modeled by stochastic differential equations for more than half a century. The stability of such equations is determined by Lyapunov functions since the study on the trajectory of Markov processes by Khasminskii [1]. On the other hand, the stochastic rolling motion of a ship in following seas in the case of a group of following steep waves encountering the ship first appears in the studies of Vinje [2] and Odabasi [3]. Vinje [2] conducts some of the stability analyses for the nonlinear, stochastically and parametrically excited motion of a ship. The conditions leading to unstable motions of a ship in narrow band random following waves with an encounter frequency close to twice of natural frequency $(\omega_e \approx 2\omega_0)$ is proposed by Odabasi [3]. Moreover, the boundary between stable and unstable regions of motion due to damping and wave steepness is first introduced by Odabasi [3]. Francescutto et al. [4] also give emphasis to the effective damping which can be used as a threshold on parametric rolling in the case of both regular and irregular waves. Influence of the initial conditions on the length of transient are presented by means of an extensive series of experiments on the parametrically excited rolling motion of a partially restrained post-panamax containership in longitudinal irregular long crested waves by Bulian et al. [5]. It is shown that the potential of prediction of parametric roll modeled by an analytical formula appears in the guidelines of ABS [6] and ITTC [7] is doubtful when the variation of righting arm (GZ) is non-harmonic by Spyrou et al. [8]. An analytical formula to determine maximum width of the safe basin of stochastic rolling motion in following seas is derived by Üçer and Söylemez [9]. Moreover, in this study the nonlinear damping is taken into account in order to investigate its effect on the stability of rolling motion. The mathematical model of a stochastic rolling motion of a ship in following seas is presented in Section 2. In Section 3, the minimum required damping moment for a stable ship in stochastic following seas is determined by using

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* Corresponding author.
E-mail address: ucerer@itu.edu.tr (E. Üçer).

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Nomenclature

$\ddot{\phi}$	roll angular acceleration
$\dot{\phi}$	roll angular velocity
ϕ	roll angle
I	virtual moment of inertia
Δ	buoyancy force
GM	initial metacentric height
GZ(ϕ)	righting arm as a function of the roll angle
δGM	maximum variation of initial metacentric height due to waves in deterministic case
ω_e	frequency of wave encounter
ω	wave Frequency
ω_e	roll natural frequency
χ	wave heading angle
K	diffusion constant of stochastic process
σ	the ratio of K to I
$\xi(t)$	Gaussian white noise
$B(\dot{\phi})$	nonlinear damping moment
B_1	linear damping coefficient
B_3	cubic nonlinear damping coefficient
t	time
U	vessel speed
g	gravitational acceleration
μ	ratio of linear damping coefficient to virtual moment of inertia (B_1/I)
η	ratio of cubic nonlinear damping coefficient to virtual moment of inertia (B_3/I)
a	ratio of maximum variation of initial metacentric height in deterministic case due to waves to initial metacentric height
gz(ϕ)	ratio of righting arm to virtual moment of inertia
c_3	cubic righting arm coefficient
ϕ_v	vanishing stability angle
ϕ_1, ϕ_2	state coordinates for roll angle and roll angular velocity respectively in Chapter 2
x	ϕ_2
$g(x)$	an odd function represents the righting moment
x_v	roots of $g(x)$
$V(x,\dot{x})$	Lyapunov function
q, y	state coordinates for roll angle and roll angular velocity respectively in Section 3
T	simulation time
Δt	time interval
N	number of points in the time interval
n	indices
W	Wiener process
ΔW_n	increment of Wiener process
κ_n	normally distributed random variables

Lyapunov function derived by Schurz [10]. In Section 4, equation of the nonlinear stochastic rolling motion for the values of damping analytically obtained in Section 3 is numerically integrated to see whether the ship is stable or not by using stochastic Euler method. From the numerical results, it can be seen that analytically obtained damping values are sufficient.

2. Mathematical model

The stochastic rolling motion of a ship in following seas is modeled as follows:

$$I\ddot{\phi} + B(\dot{\phi}) + \Delta\mathrm{GZ}(\phi) - \Delta\delta\mathrm{GM}\phi\sin\omega_e t - K\phi\xi(t) = 0 \tag{1}$$

where $\ddot{\phi}$ is the roll angular acceleration, $\dot{\phi}$ is the roll angular velocity, ϕ is the angle of roll, I is the virtual moment of inertia, Δ is the buoyancy force, GZ is the righting arm as a function of the roll angle and it can be approximated by odd polynomials, δGM is the maximum variation of initial metacentric height due to waves, ω_e is the frequency of wave encounter and determined by Eq. (2b), χ is the wave heading angle, K is the diffusion constant of stochastic process and shows the effect of the randomness of seaway on the restoring moment of the ship, $\xi(t)$ is Gaussian white noise and $(\cdot)$ denotes the derivative with respect to time t.