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Temporal and spectral responses of a softening Duffing oscillator undergoing route-to-chaos

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Because nonlinear responses are oftentimes transient and consist of complex amplitude and frequency modulations, linearization would inevitably obscure the temporal transition attributable to the nonlinear terms, thus also making all inherent nonlinear effects inconspicuous. It is shown that linearization of a softening Duffing oscillator underestimates the variation of the frequency response, thereby concealing the underlying evolution from bifurcation to chaos. In addition, Fourier analysis falls short of capturing the time evolution of the route-to-chaos and also misinterprets the corresponding response with fictitious frequencies. Instantaneous frequency along with the empirical mode decomposition is adopted to unravel the multi-components that underlie the bifurcation-to-chaos transition, while retaining the physical features of each component. Through considering time and frequency responses simultaneously, a better understanding of the particular Duffing oscillator is achieved.

1. Introduction

One of the essential objectives in studying a nonlinear system is to obtain the condition that guarantees the existence of periodic solutions so that their stabilities can be subsequently determined [1]. Steady-state solution is obtained for small but finite amplitude oscillations around the equilibrium point to estimate the threshold value of the excitation amplitude, stability region, and number of limit cycles. Linearization is performed under the assumption that if the operation range is in the immediate proximity of the equilibrium point of the nonlinear system, the response of the linearized model would approximate the nonlinear one with accuracy. However, there are cases that, although giving correct time profile of the nonlinear response, the inherent components resolved using perturbation methods neither collectively nor individually provide any physically meaningful representation of the nonlinear system [2]. Applying linearization to investigate nonlinear system without exercising proper discretions would obscure the underlying nonlinear characteristics and risk misinterpreting the stability bound.

Fourier-based analyses have been widely accepted as a tool for exploring nonlinear system. Because stationary sinusoids are employed in representing time-varying signals of inherent nonlinearity, the use of Fourier domain methodologies would also risk misrepresenting the underlying physics of the nonlinear system being investigated [3]. As most methods employed to process nonstationary signals are Fourier-based, they also suffer from the shortcomings associated with Fourier transform [4]. The fact that nonlinear responses including route-to-chaos are intrinsically transient, nonstationary with coupled amplitude–frequency modulation implies that, if a nonlinear response is to be fully characterized, the inherent amplitude modulation (AM) and frequency modulation (FM) need to be temporally decoupled [4]. The concept of instantaneous

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frequency (IF) is adopted in this study to resolve the dependency of frequency on time. Growing attention is focused on the Hilbert–Huang transform (HHT), which has been used to investigate the response of quadratic and cubic nonlinearities [5], Duffing oscillators [6], dynamic systems with slowly-varying amplitudes and phases [7], and fault induced nonlinear rotary [8]. Because HHT does not use predetermined basis functions and their orthogonality for component extraction, it provides instantaneous amplitude and frequency of the extracted components for the accurate estimation of system characteristics and nonlinearities [9]. It is shown that HHT is better appropriate than sinusoidal harmonics for characterizing nonstationary and transient responses. The interpretation of nonlinearity using IF is found to be both intuitively rigorous and physically valid.

Various Duffing oscillators have been explored to help elucidate a wide range of physical applications in the real-world. In Ref. [10] the response of a damped Duffing oscillator with harmonic excitation is analyzed by second-order perturbation solutions along with Floquet analysis to predict symmetry-breaking and period-doubling bifurcation. Duffing oscillators under nonstationary excitations are also considered by many, where linear and cyclic variations of frequencies and amplitudes are applied and nonstationary bifurcation is studied. It is shown that nonstationary process is distinct from stationary process with different characteristics [11,12]. Nonetheless, these perturbation method based studies on nonlinear systems generate nonphysical results that are bound to be misinterpreted. The presentation that follows reviews the nonlinearity and nonstationary bifurcation of a softening Duffing oscillator from the time-frequency perspective established using IF. It is noted that although IF is considered a viable tool for exploring nonlinear dynamic response, little effort has been made to study the generation and evolution of bifurcation to ultimate chaotic response, a process that is inherently nonstationary and transient. A Duffing oscillator and its linearized counterpart are studied first by fast Fourier transform (FFT), short time Fourier transform (STFT), Gabor transform, and instantaneous frequency (IF). The second part of the paper presents an in-depth investigation into the route-to-chaos generated by the Duffing oscillator under nonstationary excitation using conventional nonlinear dynamic analysis tools and IF.

2. Instantaneous frequency and intrinsic mode function [3]

The concept of instantaneous frequency was introduced to resolve the time evolution of the spectral response of a nonstationary signal [14] – a task of which Fourier-based analyses fall short. IF is defined as the time derivative of the phase of a complex signal. Such a definition was shown to work well with signals of monocomponent. In the following the definition of instantaneous frequency is briefly reviewed. A time-varying signal $r(t)$ having both amplitude modulated (AM) and frequency modulated (FM) components can be represented as $r(t) = c(t)\cos(\theta(t))$. Its analytic signal is

$$s(t) = w(t) + iz(t) = w(t) + iH(t) = c(t)\exp(i\theta(t)) \tag{1}$$

where $s(t)$ is the analytic signal, $c(t)$ is the instantaneous amplitude, $\theta(t)$ is the instantaneous phase, and $z(t)$ is the imaginary part of $s(t)$. Defining $H(t)$ as the Hilbert transform of the time varying signal $w(t)$

$$z(t) = H[w(t)] = \frac{p}{\pi}\int_{-\infty}^{\infty}\frac{w(\tau)}{t-\tau}d\tau = w(t)\cdot(p/\pi t) \tag{2}$$

with p being the Cauchy principle value. In theory $w(t)$ and $z(t)$ are out of phase by $\pi/2$. The instantaneous amplitude and phase are defined as $c(t) = \sqrt{w^2(t) + z^2(t)}$ and $\theta(t) = \tan^{-1}(z(t)/w(t))$, respectively. By Ville's definition [14] the derivative of the instantaneous phase is the instantaneous frequency, thus, $f(t) = (1/2\pi)(d\theta(t)/dt)$. Such a definition agrees with our intuition for instantaneous frequency and captures the concept of instantaneity in nature. However, the definition fails when applied to multicomponent signals for the reason that it adversely averages all the individual IFs associated with each individual monocomponent and interprets them as single instantaneous frequency. In addition to falling short on providing a unified interpretation for signals of multicomponent, the definition also allows infinite and negative frequencies to be induced. The empirical mode decomposition (EMD) scheme proposed by Huang et al. [2] effectively decomposes a time series into its several inherent physical modes of motion called the Intrinsic Mode Functions (IMF). Each IMF is an orthogonal set of intrinsic monocomponent from the response and retains the inherent physical features. By definition, every mode has the same numbers of extrema and zero crossings and the inherent oscillation is symmetric with respect to a local mean defined by the average of the maximum envelope and minimum envelop without resorting to any time scale. All the inherent IMFs, $C_1(t)$, $C_2(t)$, $C_3(t)\ldots$, and C_m of the dynamic response $s(t)$ can be extracted using a shifting algorithm that resolves a residual term $R(t)$ that carries no frequency component. It can be shown that the summation of all the IMFs and the residual term restore back to the response, $s(t) = \sum_{j=1}^{m} C_j(t) + R(t)$. From the decomposition process, it is understood that the first mode (C_1) has the smallest time scale, indicating that it includes the highest frequency components. As the decomposition continues, the frequency components included in IMF become lower. The decomposition is based on the local characteristic time scale of the data to produce an adaptive basis and does not employ a set of fixed time scales.

Marginal spectrum, defined below in Eq. (3), provides a quantitative measurement of the cumulated weight of all the instantaneous frequency components over a specific time period,

$$f_{01}(\omega) = \int_{t0}^{t1} F(\omega, t)dt \tag{3}$$