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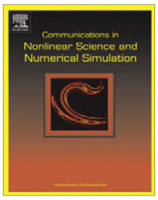

Nonlinear Cournot oligopoly games with isoelastic demand function: The effects of different behavior rules

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A R T I C L E I N F O

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A B S T R A C T

The analysis of asymptotical convergence for the oligopoly game has always been important to characterize the firms' long-term behavior. In the nonlinear oligopoly competition possibly involving chaotic fluctuations, non-convergent trajectories are particularly undesirable since the resulting behavior will become unpredictable. In this paper, consistent with a traditional assumption that the firms update their outputs simultaneously, we at first construct an adjustment process and discuss the convergence to the equilibrium for a nonlinear Cournot duopoly game with the isoelastic demand function. We indicate that the tendency to instability does rise with the number of firms and the adjustment speeds. In particular, we alter this assumption from simultaneous decisions to sequential decisions so that the latter firms are able to observe the former ones at every time periods. We finally arrive at a conclusion that the unique equilibrium is convergent as long as the adjustment speeds are less than a fixed threshold, no matter what the number of the firms. Our findings show that the firms with sequential decisions can achieve the equilibrium more easily.

1. Introduction

The first adjustment process for the oligopoly game is introduced by Cournot [5], where the firms make their decisions simultaneously and the rivals' outputs are assumed the same as in the immediately previous period. Following this assumption, Theocharis [21] argues that the equilibrium will become unstable for more than three firms in a simple Cournot oligopoly game with linear cost functions. By introducing nonlinear cost functions and adjustment speeds, Fisher [7] shows that increasing marginal costs and decreasing adjustment speeds act as stabilizing factors.

Cournot's hypothesis avoids the rigorous assumption of perfect information and complete rationality, but it is incompletely consistent with the actual decision-making process yet since the rivals' outputs usually change until the convergent equilibrium is eventually reached. There has been much literature concerning how to overcome this myopic hypothesis by introducing different learning mechanisms.

One crucial learning mechanism has been investigated widely, by which the expectations for rivals' quantities are formed adaptively. For example, introducing this mechanism, Okuguchi [13] derives a sufficient condition for the stability of a continuous adjustment process in a Cournot oligopoly game, where each oligopolist updates its actual output by a sign-preserving, monotonic function with respect to the difference between profit maximizing output and actual one. Okuguchi [15] then considers alternative sophisticated adaptive expectations to discuss the stability of oligopoly equilibrium under a continuous adjustment dynamic in a Cournot oligopoly model. Okuguchi [14] also investigates a Bertrand oligopoly game by assuming

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that the price expectations are formed adaptively. Szidarovszky et al. [19] provide some stability analysis of a discrete adjustment process in a linear Cournot oligopoly game with adaptive expectations.

Extrapolative expectations are taken into account in oligopoly competition and other economic models such as evolutionary game dynamics, where the expectations for rivals' outputs can be estimated by the derivative means. Motivated by the work of Enthoven et al. [6] as well as Arrow and Nerlove [1], Quandt [18] constructs a price adjusting oligopoly with this type of expectations and obtains a weaker sufficient condition for stability under a continuous adjustment process. By introducing such analogous strategy update means for two important classes of evolutionary game dynamics, the fictitious dynamic and the gradient dynamic, Shamma and Arslan [20] show convergence to Nash equilibrium can always be achieved in the ideal case of exact derivative measurements.

Also, another class of expectations has been introduced in the oligopoly game and other adjustment processes so that each firm has simple forecasting with respect to its rivals' outputs. Holt [9] indicates that if each oligopolist adopts forecasting functions to estimate its rival's choices, the instability will disappear, especially when the cost and demand structures are quadratic in a Cournot duopoly game. Kamalinejad et al. [10] show that a stable Cournot oligopolistic market can be always guaranteed in case the rivals' choices are estimated by linear regression and recursive weighted least-squares method. Under a gradient-like decisional process for boundedly rational consumers, Naimzada and Tramontana [12] point out their consumption choices may converge to the equilibrium provided that they become able to learn from their history by a simple least squared learning mechanism.

Even though there are many learning mechanisms proposed to relax Cournot's hypothesis, all these above are discussed based on simultaneous decisions. Following Cournot's hypothesis, in this paper we firstly construct a discrete adjustment process in a nonlinear Cournot oligopoly game with isoelastic demand function where simultaneous decisions are considered. Stability analysis shows that increasing the number of firms and adjustment speeds acts as a destabilizing factor. We then alter Cournot's hypothesis by assuming all firms make their decisions sequentially so that the latter firms are able to observe the former's newly-adjusted outputs. We finally demonstrate that corresponding revised adjustment dynamic is always stable provided that the adjustment speeds do not exceed a fixed threshold, no matter what the number of the firms. Compared with the linear Cournot oligopoly game, the dynamic properties for the nonlinear ones are more complex since chaotic fluctuations may emerge when the nonlinearity becomes strong. We show that sometimes chaos emerging in our nonlinear adjustment process with simultaneous decisions, can be eliminated as long as the sequential decisions are introduced.

This paper is different from our previous work in two key aspects [8]. For one thing, in this paper we characterize each firm's adjustment process by comparing the reaction function and the adjusted quantity, which is far different from the adjustment dynamic in [8] where each firm determines its quantity by the marginal profit. For another, the isoelastic demand function fails to satisfy some necessary assumptions [2], and therefore similar with the work in [3,11,17], it is of interest to discuss Cournot oligopoly games with this particular market structure.

The remaining of this paper is constructed as follows. In Section 2, we discuss a discrete adjustment process in a nonlinear Cournot oligopoly game with the isoelastic demand function, where the decisions are made simultaneously. We employ alternative assumption with sequential decisions to derive a revised adjustment dynamic in Section 3. Section 4 concludes this paper.

2. Simultaneous decisions

We consider a Cournot oligopoly market where n firms produce homogeneous goods at discrete time periods. In order to determine the corresponding profit-maximizing quantity, every firm at each time period must form an expectation for the rival's quantities in the subsequent period. Letting $q_i(t)$ be the quantity at discrete time period t, how to determine the optimal quantity (namely the reaction function) for firm i at time period $t+1$ can be solved by the optimization problem $\arg\max_{q_i}\pi_i\left(q_1^e(t+1),\ldots,q_{i-1}^e(t+1),q_i,q_{i+1}^e(t+1),\ldots,q_n^e(t+1)\right)$, where $\pi_i(\cdot)$ denotes the profit for firm i and $q_j^e(t+1)$ $(j\neq i)$ are its expectations for the competitors' outputs [4,10].

Cournot's hypothesis shows in every time period each firm determines its profit-maximizing quantity by assuming that its rivals' outputs remain the same as in the immediately previous period. That is, firm i's expectations for its competitors' outputs are assumed equal to these in the immediately previous period, $q_j^e(t+1)=q_j(t)$ $(j\neq i)$ [4,11,22]. The reaction function for firm i becomes $q_i^*(t)=\arg\max_{q_i}\pi_i(q_1(t),\ldots,q_{i-1}(t),q_i,q_{i+1}(t),\ldots,q_n(t))$.

The isoelastic demand function at time period t, i.e. the market price $p(t)$ [3,11,16,17], is reciprocal to the total demand $\sum_{i=1}^{n}q_i(t)=q_i(t)+Q_i(t)$, $p(t)=1/(q_i(t)+Q_i(t))$, where $Q_i(t)=\sum_{j=1,j\neq i}^{n}q_j(t)$.

Assuming $c_i>0$ is the marginal cost for firm i, we can obtain the expected profit

$$\pi_i(t)=q_i(t)/(q_i(t)+Q_i(t))-c_iq_i(t).$$

Maximizing this profit yields the optimal quantity for firm i

$$q_i^*(t)=\sqrt{Q_i(t)/c_i}-Q_i(t),$$

We employ a general adjustment process in this paper