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A constitutive modelling framework featuring two scales of behaviour: Fundamentals and applications to quasi-brittle failure

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ABSTRACT

We propose a constitutive modelling framework with enhanced kinematics to capture localised mode of deformation. The total strain is decomposed into two components to reflect an inelastic localisation band embedded in an elastic bulk. This is the usual case in numerical analysis of localised failure in geomaterials, when the size of the localisation band is very small compared to an element of the discretised domain under consideration. The proposed framework takes into account the sizes and corresponding behaviours of the two inelastic and elastic zones and hence gives derived constitutive models a length scale. This is an essential feature in dealing with size effect issues as a consequence of localised failure in geomaterials. The proposed framework is applied to a constitutive model for the failure analysis of quasi-brittle materials. The implementation algorithms are developed and novel features are illustrated through numerical examples.

1. Introduction

Localised failure in geomaterials involves the inelastic loading inside a very thin, but finite thickness, band, while the material outside the band usually unloads elastically. The dissipation properties of the material are therefore fully governed by what actually happens inside this very thin localisation zone. The thickness of this band is related to the microstructure of the material and is a physical quantity that brings size effect features to the material response [1,2]. Shear/compaction/dilation bands observed in soils, porous rocks, shear bands in metallic materials and Fracture Process Zone (FPZ) in quasi-brittle materials are typical examples of this localised failure. In such cases the homogeneity of the continuum is lost, with discontinuity of kinematic fields across the boundary of the localisation band. This means the definitions of stress and strain over a volume element containing both the bulk elastic and an embedded inelastic localisation zone are not physically meaningful anymore. Given the above aspects of localised failure, it is essential that constitutive models take all of them into account by representing the size and behaviour of the material inside the localisation band as well as those of the surrounding bulk elasticity. Failure to do so, which is the case in most classical constitutive models, leads to the loss of physical meaning of the macro stress and strain, as these quantities in such cases are defined over a non-homogenous volume element.

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Nomenclature

G_F	mode I fracture energy
g_F	specific fracture energy; $g_F = G_F/h$
h	effective width of the fracture process zone
H	effective size of the spatial discretisation
f	volume fraction
L_c	critical length
f_t'	uniaxial tensile strength
E	Young modulus
ν	Poisson's ratio
$\mathbf{a}_o$	isotropic elastic stiffness tensor of the bulk material
$\mathbf{a}_i$	isotropic elastic stiffness tensor of the material inside the localisation zone
$\mathbf{a}_i^T$	tangent stiffness tensor of the material inside the localisation zone
$\mathbf{D}_i$	isotropic elastic compliance tensor of the material inside the localisation zone
$\mathbf{A}_o$	acoustic tensor associated with $\mathbf{a}_o$
$\mathbf{A}_i$	acoustic tensor associated with $\mathbf{a}_i$
$\mathbf{A}_i^T$	acoustic tensor associated with $\mathbf{a}_i{}^T$
$\mathbf{K}_i$	elastic stiffness of the localisation zone idealised as a surface
$\mathbf{K}_i^T$	tangent stiffness of the localisation zone idealised as a surface
$\mathbf{n}$	normal vector of the localisation band
Ω_o	volume of the material outside the localisation zone
Ω_i	volume of the material inside the localisation zone
Ω	total volume of the sub-domain; $\Omega = \Omega_i + \Omega_o$
D	scalar damage variable
$[\mathbf{u}]$	displacement jump
$\boldsymbol{\varepsilon}$	total average strain
$\boldsymbol{\varepsilon}_i$	total strain of the material inside the localisation zone
$\boldsymbol{\varepsilon}_o$	total strain of the material outside the localisation zone
$\boldsymbol{\varepsilon}_i^p$	plastic strain of the material inside the localisation zone
$\boldsymbol{\varepsilon}_i^e$	elastic strain of the material inside the localisation zone
$\boldsymbol{\sigma}$	average stress tensor
$\boldsymbol{\sigma}^+$	"positive" part of the stress tensor $\boldsymbol{\sigma}$ from the eigenvalue decomposition
$\boldsymbol{\sigma}_i$	stress tensor of the material inside the localisation zone
$\boldsymbol{\sigma}_o$	stress tensor of the material outside the localisation zone
σ^m	the mth principal stress
$\mathbf{t}_i$	traction at the boundary of the localisation zone
$\mathbf{p}^m$	the unit vector of the mth principal direction
$\mathbf{Q}$	tensor defined by the ratio between plastic strain rate and damage rate
Ψ	Helmholtz free energy potential of a unit volume
g_i	Gibbs free energy potential for a unit volume inside the localisation zone
Ψ_i	Helmholtz free energy potential for a unit volume inside the localisation zone
Ψ_o	Helmholtz free energy potential for a unit volume outside the localisation zone
$\underset{\sim}{\Psi}_\Gamma$	Helmholtz free energy potential for the localisation zone idealised as a surface
$\underset{\sim}{\Phi}$	dissipation potential (total rate of dissipation) for a unit volume
$\underset{\sim}{\Phi}_i$	dissipation potential for a unit volume inside the localisation zone
Φ_o	dissipation potential for a unit volume outside the localisation zone
ϕ_D	damage component defining the explicit form of the dissipation potential
ϕ_p	plasticity component defining the explicit form of the dissipation potential
Φ_D	damage dissipation (damage rate of dissipation)
Φ_p	plastic dissipation (plastic rate of dissipation)
χ_D	damage energy (thermodynamically associated with the damage variable D)
$F^*(D, \boldsymbol{\sigma}_i)$	function controlling the overall rate of dissipation
$F(D)$	function controlling the evolution of damage process
ω	parameter controlling the partition of fracture energy
E_p	parameter controlling the initial slope of the softening stress–strain curve
n	parameter controlling the slope change of the softening stress–strain curve
$\delta\lambda$	non-negative plasticity-like multiplier
y	loading function, or combined damage-yield function in stress/strain space
y^*	loading function, or combined damage-yield function in mixed energy–stress space
d_{50}	median diameter for which half of the sample is finer
$d_{\max}$	maximum aggregate diameter