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Lattice hydrodynamic model of pedestrian flow considering the asymmetric effect

Xingli Li [a], Hua Kuang [b,c,*], Yanhong Fan [b]

[a] School of Applied Science, Taiyuan University of Science and Technology, Taiyuan 030024, China
[b] College of Physical Science and Technology, Guangxi Normal University, Guilin 541004, China
[c] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China

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ABSTRACT

The original lattice hydrodynamic model of traffic flow is extended to single-file pedestrian movement at middle and high density by considering asymmetric interaction (i.e., attractive force and repulsive force). A new optimal velocity function is introduced to depict the complex behaviors of pedestrian movement. The stability condition of this model is obtained by using the linear stability theory. It is shown that the modified optimal velocity function has a remarkable influence on the neutral stability curve and the pedestrian phase transitions. The modified Korteweg–de Vries (mKdV) equation near the critical point is derived by applying the reductive perturbation method, and its kink–antikink soliton solution can better describe the stop-and-go phenomenon of pedestrian flow. From the density profiles, it can be found that the asymmetric interaction is more efficient than the symmetric interaction in suppressing the pedestrian jam. The numerical results are consistent with the theoretical analysis.

1. Introduction

In recent years, pedestrian flow has attracted considerable attention in the field of physical science and engineering [1–3]. On the one hand, the complex behaviors of pedestrian can result in many interesting non-linear phenomena and collective behaviors, such as jamming and clogging, lane formation and oscillations at bottlenecks in counter flow, sudden transitions from laminar to stop-and-go and "turbulent" flows [4–6]. On the other hand, to gain an insight into these complex behaviors is of vital importance in the management and optimization of pedestrian facilities, especially to avoid and relieve pedestrian congestion.

A lot of typical pedestrian flows have been simulated with various models, such as the social force models [7,8], the hydrodynamic models [9,10], the cellular automaton models [11–13], the lattice gas models [14–16]. However, for the pedestrian flow at middle and high density, the related research are scarce. How to describe and analyze its dynamic characteristics qualitatively, especially, stop-and-go phenomenon, is an interesting but still open problem.

Because the pedestrian system is very analogous to the vehicle traffic flow in many respects [17], it is feasible for us to investigate pedestrian flow using the enlightenment and reference of traffic model. However, the particularity of pedestrians, i.e., independence, randomness and activity, etc. must be taken fully into account. The lattice hydrodynamic model, being convenient for analyzing density waves in traffic flow, has been receiving increasing attention from researchers [18–20]. Recently, using the lattice hydrodynamic traffic model, Xue et al. studied the jamming transitions and density waves

* Corresponding author Address: College of Physical Science and Technology, Guangxi Normal University, No. 15, Yucai Road, Guilin 541004, China.
E-mail address: khphy@shu.edu.cn (H. Kuang).

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analytically in bidirectional pedestrian traffic [21,22], in which the optimal velocity function, often chosen as a simple hyperbolic-tangent function of the headway (or density), depicts the simple behavioral process of a walker, similar to that in traffic flow. The principal feature of the function is that its first order derivative is symmetric about the point of inflexion on its curve, which reflects indirectly an unrealistic scenario where the acceleration (attractive effect) and deceleration (repulsive effect) mechanisms of walkers are same in movement process relative to "equilibrium state". However, real pedestrian flow shows that a walker's response is completely asymmetric for making a decision to accelerate or decelerate after he realizes the change of distance ahead of him. In fact, the response to deceleration is more sensitive for a walker than that to acceleration. It is also a very crucial condition for safe walking to avoid collision with others.

As far as we know, in traffic flow, the asymmetry between acceleration and deceleration is considered through distinguishing the difference of the sensitivity [23,24]. Nevertheless, differences of sensitivities among pedestrians are generally small [25] and the above method is not applicable for describing the real pedestrian flow. Therefore, it is very necessary to explore a new way to solve this open problem. In this paper, through modifying the general optimal velocity function, we investigate how this asymmetric mechanism influences pedestrian movement.

The paper is organized as follows: A new optimal velocity function is introduced through considering the asymmetric interaction in a single-file pedestrian movement in Section 2. In Sections 3 and 4, the linear and nonlinear stability analysis are carried out, respectively. In Section 5, numerical simulations are performed and the intrinsic mechanism of the corresponding phase transition is explored. Finally, the obtained results are summarized.

2. Model

The one-dimensional lattice hydrodynamic model of traffic is extended to the pedestrian flow in a circular route or a single-file movement. The pedestrian is described by the following differential-difference equations:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0 \tag{1}$$

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}(t)) \tag{2}$$

where the subscript j indicates j site on the one-dimensional lattice; $\rho_j(t)$ and $v_j(t)$ represent the density and velocity at site j at time t, respectively; ρ_0 is the average density; $\tau = 1/a$ is introduced to denote the delay time with which the walker's velocity reaches the optimal velocity as the pedestrian flow is varying; V is called the optimal velocity function. Eliminating velocity terms in Eqs. (1) and (2), we obtain the density equation

$$\partial_t \rho_j(t+\tau) + \rho_0^2[V(\rho_{j+1}(t)) - V(\rho_j(t))] = 0 \tag{3}$$

Generally, the optimal velocity function has the following properties [18]: It is a monotonically decreasing function with the increase of density and has an upper bound (i.e., the maximal velocity). Also, it is important that it has one turning point at least. The optimal velocity function is usually selected as follows [20]:

$$V(\rho_j(t)) = \tanh\left(\frac{2}{\rho_0} - \frac{\rho_j(t)}{\rho_0^2} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right) \tag{4}$$

where ρ_c is the critical density and ρ_0 is the average density.

As we pointed out in Section 1, the optimal velocity function (4) could not describe the realistic behavior of walkers due to the symmetric interaction. Based on the above consideration, we choose the following optimal velocity function which can describe the "attractive force" and "repulsive force" between walkers as well as meet the properties mentioned:

$$V(\rho_j(t)) = V_0 + V_F = V_0 + \left(\frac{A + \gamma\rho_j(t)^\beta}{B + \alpha\rho_j(t)^\beta}\right) \tag{5}$$

where V_0 is a constant expressing "equilibrium velocity", which means pedestrians neither accelerate nor decelerate (i.e., no interaction between pedestrians or the resultant of "attractive force" and "repulsive force" is zero). V_F is called as "force", in which A, B, α, γ and β are undetermined parameters which should meet the varying tendency of $V(\rho_j)$ as mentioned above and ensure the same dimension between V_0 and V_F. The form of Eq. (5) is introduced by taking into account asymmetric effect, with Eq. (3), named as model 2. The original model described by Eqs. (4) and (3) is named as model 1, which reflects the symmetric interaction between walkers. Here, note that $|V_F(\rho_j)|$ denotes the magnitude of interaction and the sign of $V_F(\rho_j)$ only means the direction of the interaction forces, i.e., if $V_F(\rho_j) > 0$, the interaction is attractive, and if $V_F(\rho_j) < 0$, the interaction is repulsive (see Fig. 1).

Based on the above analysis and realistic mathematical and physical consideration, the reasonable parameters in the modified optimal velocity function are suggested as $A = 0.9$, $B = 1$, $\alpha = 18$, $\beta = 3$, $\gamma = -19.8$ and $V_0 = 1$. In the original optimal velocity function (4), $\rho_0 = \rho_c = 0.4$ is selected. The corresponding curves of optimal velocity function $V(\rho_j)$ are shown in Fig. 1. Compared with the hyperbolic-tangent function, the modified optimal velocity function is not symmetric and the velocity is not zero at high density, which can describe the real pedestrian better than tanh-function does. Here it should be noted that when $\rho_0 = \rho_c = 0.4$ is selected, the optimal velocity (4) is equal to negative for $0.8 < \rho < 1$, but approaching zero. For the single-file pedestrian movement at the middle and high density under the periodic boundary condition, for safety reasons,